Maximum Lifetime of Sensor Networks with Adjustable Sensing Range

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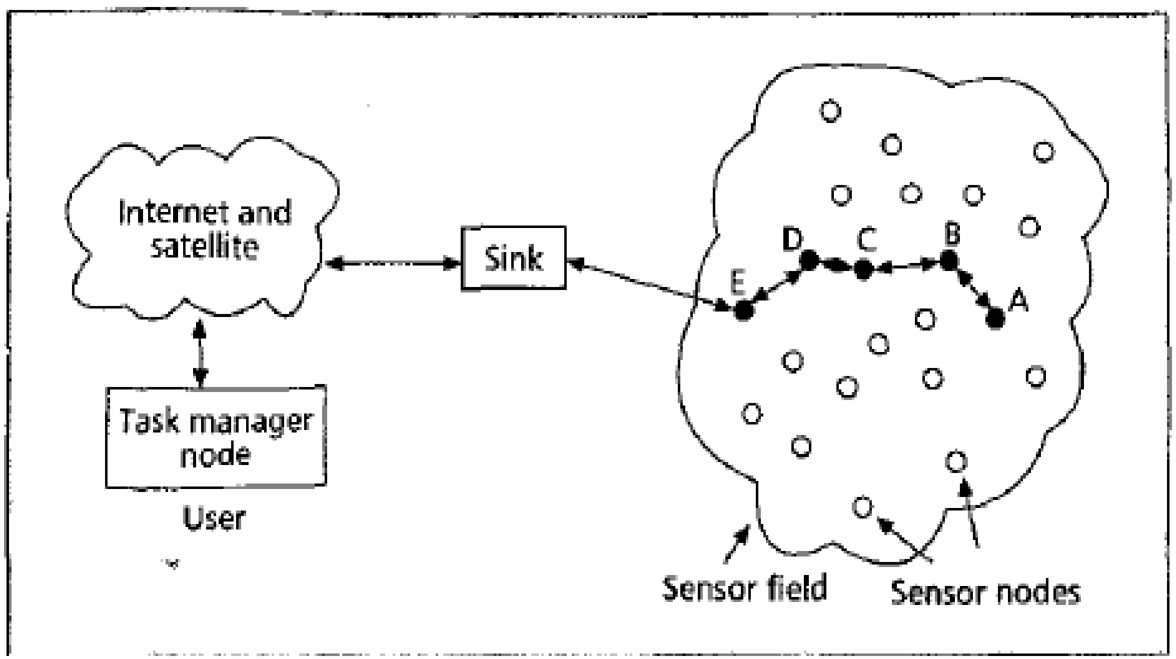
Outline

- Background
- Problem Statement and LP formulation
- The approximation algorithm
- Greedy solution to the dual problem
- Experimental Evaluation
- Discussion
- Conclusion

Introduction

- Sensor networks
- Major constraints – energy, computation, bandwidth



Introduction

- High node density implies that only a subset of nodes need to be active.
- *Target coverage problem* – A set of targets that need to be covered.
- *Idea:* Pick a set of active sensors as a number of set covers C_1, C_2, ..., C_m and use these one by one
- *Question:* How long? Need to assign a *time* to each cover. Pairs (C_m, t_m)

Adjustable range model

- Now lets make things more interesting…
- *Adjustable range* – Each sensor can vary its range from 0 (off) to *MAXDIST*
- So in addition to picking the sensors s_i that participate in (C_m, t_m) we need to associate a range r_i with each s_i
- Makes the problem more interesting because as range increases, target coverage increases but so does energy

Contributions

- Problem studied first by *Cardei et al [4]*
- We propose a different LP formulation
- Give a *provably* good heuristic
- Can handle non-uniform battery at each sensor
- Smooth sensing range model in place of discrete range model

Related work

- Cardei et al. [4]
- Maximize number of subsets – limit k

- c_k, boolean variable, for $k = 1..K$; $c_k = 1$ if this subset is a set cover, otherwise $c_k = 0$.
- x_{ikp}, boolean variable, for $i = 1..N$, $k = 1..K$, $p = 1..P$; $x_{ikp} = 1$ if sensor i with range r_p is in cover k, otherwise $x_{ikp} = 0$.

i: i^{th} sensor, when used as index
j: j^{th} target, when used as index
p: p^{th} sensing range, when used as index
k: k^{th} cover, when used as index

Maximize $c_1 + ... + c_K$
subject to

$\sum_{k=1}^{K}(\sum_{p=1}^{P} x_{ikp}e_p) \leq E$ for all $i = 1..N$

$\sum_{p=1}^{P} x_{ikp} \leq c_k$ for all $i = 1..N$, $k = 1..K$

$\sum_{i=1}^{N}(\sum_{p=1}^{P} x_{ikp} * a_{ipj}) \geq c_k$ for all $k = 1..K$, $j = 1..M$

$x_{ikp} \in \{0, 1\}$ and $c_k \in \{0, 1\}$

Sensor Network Lifetime Problem (SNLP) with range assignment

- Given a monitored region R, a set of sensors s_1, s_2, .. s_m and a set of targets $i_1, i_2, \ldots i_n$, and energy supply b_i for each sensor, find a monitoring schedule (C_1,t_1), …, (C_k,t_k) *and* a range assignment for each sensor in a set C_i such that:

(1) $t_1+\ldots+t_k$ is maximized,

(2) each set cover monitors all targets $i_1,\ldots,i_n$ and,

(3) each sensor s_i does not appear in the sets $C_1,..C_k$ for a energy more than b_i

LP formulation

$$\textit{Maximize:} \sum_{j=1}^{m} t_j$$

$$\textit{Subject to} \sum_{j=1}^{m} C_{ij}\, t_j \leq b_i \qquad (1)$$

where,

b_i is the battery for sensor *i,*

Rows i, i=1, …, n represent each sensor,

Columns j, j=1, …, m represent each sensor cover,

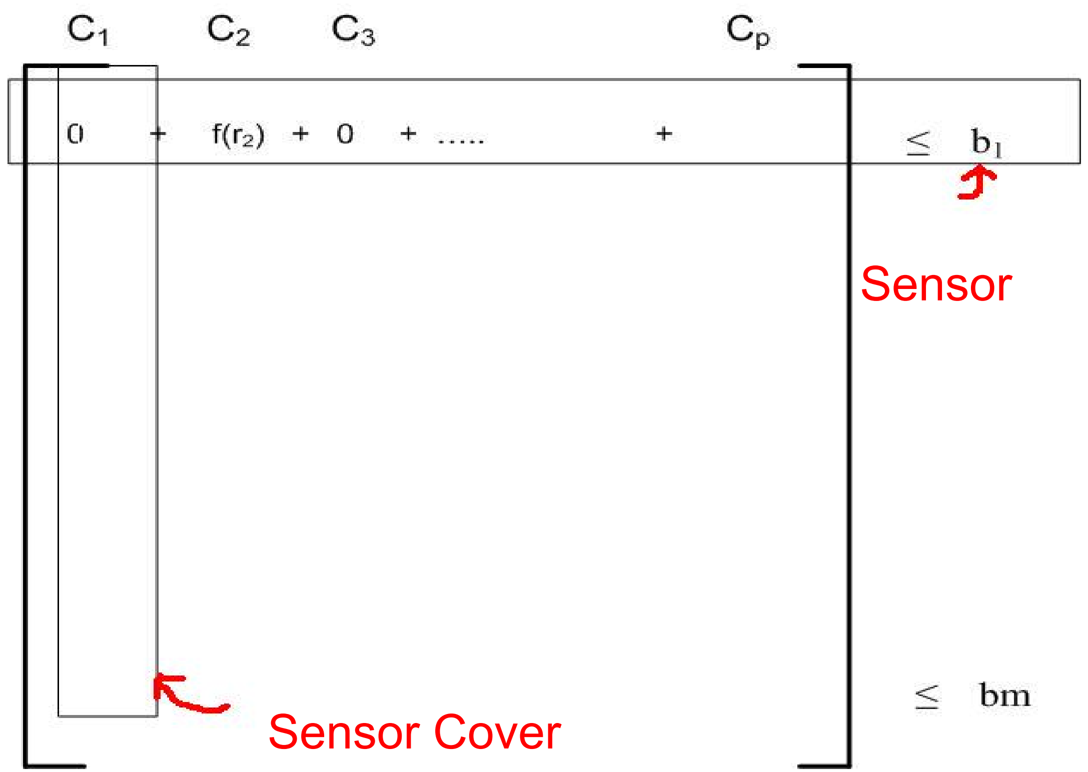
and, $C_{ij} = 0$ if sensor i is not in sensor cover j,

$C_{ij} = g(d)$, if sensor i is in sensor cover j with a sensing range fixed to d and g is a function of energy over distance.

Example

- Suppose *m* sensors, *p* covers
- *Maximize:* $\sum^{p} tj$

Subject to:



Comments

- Substantially different from formulation in *[4]* (Max . $C_1 + C_2 + \ldots + C_k$)
- They indirectly maximize number of sets up to some limit *k*. We directly maximize lifetime *t*
- Also, it can be shown that having more than *n* covers C_j with non-zero t_j is of no use, where *n* is the order of sensors
- Problem: Exponential columns in *n*

Garg-Könemann

- Defn 1 – Packing LP.
- General form:

$$max\{c^T x \mid Ax \leq b, x \geq 0\} \qquad [8]$$

where, A, b and c are (m x n), (m x 1) and (n x 1) matrices whose entries are positive.

- GK needs an *f*-approximation to find the minimizing length column of A
- $length_y(j) = \sum_i A(i,j)\, y(i) / c(j)$ for any positive vector y

The Generic GK algorithm

- While (true) {

  Use *f-approximation* to find a good set cover

  Update the weight

  If (exit condition)   break;

  }
- Exit condition:
  - Error condition is matched
  - Enough set covers

Result

- THEOREM 4.1. *The Lifetime problem with adjustable sensing range assignment can be approximated within a factor of* $(1+\epsilon) f$, *for any* $\epsilon > 0$ *by using the Algorithm of Fig. 1, where* f *is the approximation ratio of the algorithm that picks the minimum weight column in Fig. 1.*

  This result is implied by the Garg-Könemann algorithm [8].

- So we need an *f-approximation* to the dual problem

Minimum Weight Sensor Cover with Adjustable Sensing Range

- Given a monitored region R, a set of sensors s_1, s_2, …, s_n and a set of targets covered by each sensor for a range r_i and the weight w_i for each sensor, find the sensor cover with minimum total weight.
- So the range influences the weight
- *Basic idea:* A sensor wants the best ratio of targets covered to energy spent

f-approximation function

- Greedily adding to sensor cover the sensor s_i having maximum *gval* (greedy value)

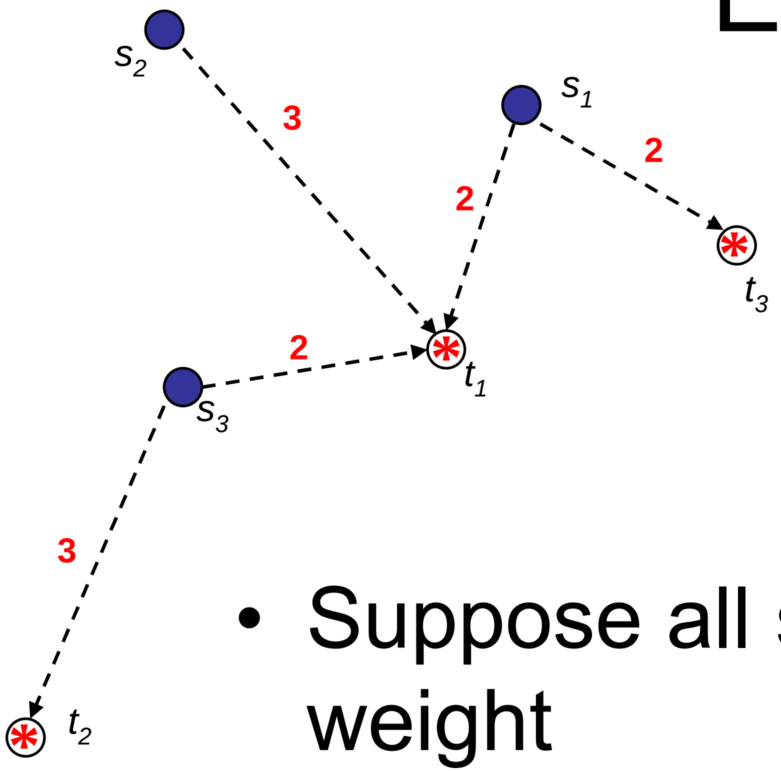
$$gval(i) = \frac{number\ of\ uncovered\ targets\ that\ s_i\ cover}{weight_i \times e_{ij}}$$

- where *weight* is packing LP variable and will be changed by Garg-Konemann algorithm
- e_{ij} is function of d_{ij} : $e_{ij} = f(d_{ij})$, *f* could be linear, quadratic or anything else
- Each sensor maintain an matrix $D_i = \left[\frac{1}{e_{i1}}, \ldots, \frac{m_0}{e_{im_o}}\right]$ and Ds_i is maximum element
- → $gval(i) = Ds_i / weight_i$

f-approximation function

- Empty set cover C.
- Repeat
  - For each sensor s_l, calculate its *gval(i)* from D_i matrix
  - Add sensor s_i having maximum *gval* value in to the set cover C.
  - other sensors update its D_i .
- Until all targets have been covered

Example



- $D_1=[2/2]$
- $D_2=[1/3]$
- $D_3=[1/2;\ 2/3]$ → $Ds_3=2/3$

- Suppose all sensor have the same weight
  - *1st pass:* $Ds_1=2/2$; $Ds_3=2/3$; $Ds_2=1/3$ → Choose sensor 1
  - *2nd pass:* $Ds_3=1/3$; $Ds_2=0$ → Choose sensor 3
  - Done: $C=\{(s_1,2), (s_3,3)\}$

Approximation Ratios

- THEOREM 5.1. *The Greedy Algorithm for the Minimum Weight Sensor Cover Problem with Adjustable Sensing Ranges has an approximation ratio $(1+\ln k)$.*

  This is from the standard greedy algorithm for the Minimum Weight Set Cover Problem with *k* points to cover.

  COROLLARY 5.2. *The Lifetime problem with adjustable sensing range assignment can be approximated within a factor of $(1+\epsilon)(1+\ln m)$ for any $\epsilon > 0$ by using the Algorithm of Fig. 1.*

  This result comes from Theorem 4.1 and Theorem 5.1 with $k=O(m)$ elements to cover, *m* being the number of targets.

Experimental Results

- 100mx100m area
- Number of Sensors N: 80 to 200
- Number of targets: 25 or 50
- Range *r* : 5m to 60m
- Same as [4] but we allow range to vary smoothly instead of discrete steps
- Same energy models – linear and quadratic
- Use GK to find sensor covers. Then solve LP for assigning time to each sensor cover.

Results

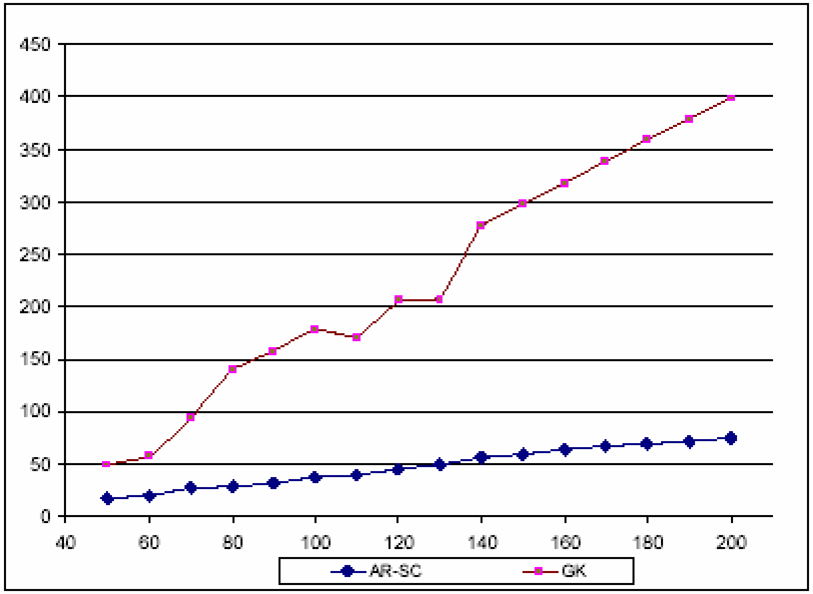


Fig 3. Variation in Network Lifetime with Number of Sensors. Number of Targets=25, Energy model is linear. AR-SC denotes the algorithm in [4]

Results

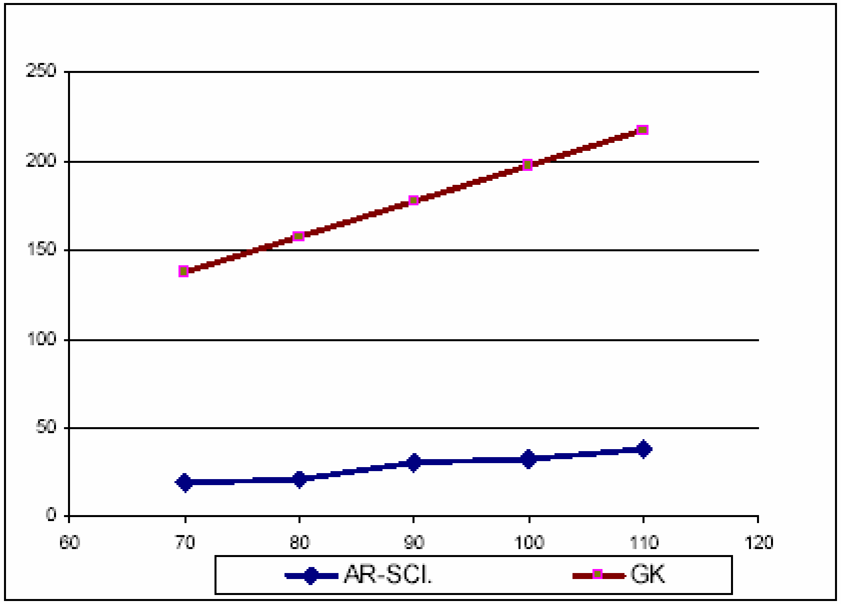


Fig 4. Variation in Network Lifetime with Number of Sensors. Number of Targets=50, Energy model is linear.

Results

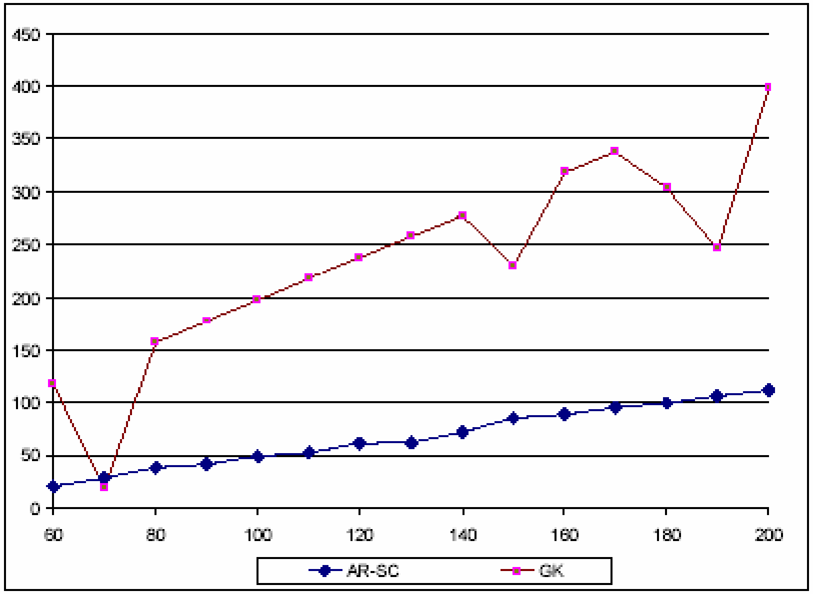
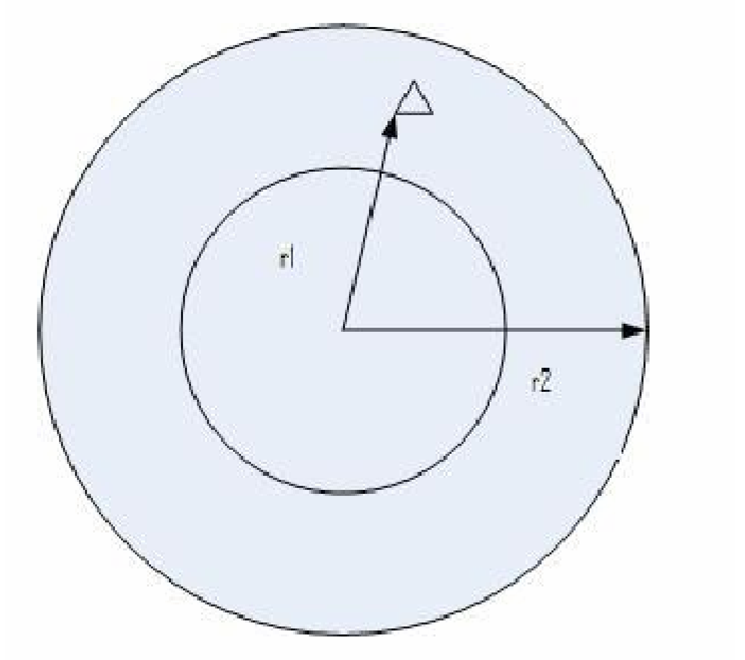


Fig 5. Variation in Network Lifetime with Number of Sensors. Number of Targets=25, Energy model is quadratic.

Reasons for improvement

- Smoothly varying sensing range
- Hence, we spend energy needed to reach target and not the next step



Reasons for improvement

- Ability to assign fractional time to each cover
- Provably good algorithm with approximation ration *(1+ln m)*

Conclusions

- New formulation
- Provably good heuristic
- Initial results indicate significant improvement
- Future work – more comparisons, distributed algorithm for the same problem