Maximal Independent Set and Minimum Connected Dominating Set in Unit Disk Graphs

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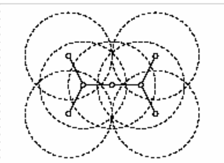
Outline

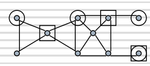
- Introduction
- Preliminaries
- Main Results
- Discussion

Introduction



A Unit Disk Graph (UDG) is an intersection graphs of circles of unit radius in the plane. An edge exists between two nodes *u* and *v* if and only if $|uv| \leq 1$

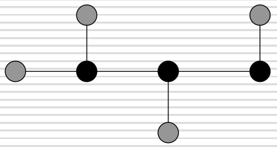
Introduction



Maximal Independent Set (MIS) is a maximal set of pair-wise non-adjacent vertices

Introduction

A connected dominating set (CDS) of a graph is a subset of the nodes such that it forms a dominating set in the graph and the subgraph induced is connected.



Computing a minimum CDS is NP-hard.

Introduction

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- J. Wu and H. Li, 'On Calculating Connected Dominating Set for Efficient Routing in Ad Hoc Wireless Networks', Proc. of the Third International Workshop on Discrete Algorithms and Methods for Mobile Computing and Communications, Page 7-14 , Aug. 1999.
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- M. Cadei, X. Cheng and D.-Z. Du, 'Connected Domination in Ad Hoc Wireless Networks,' Proc. 6th International Conference on Computer Science and Informatics, 2002.
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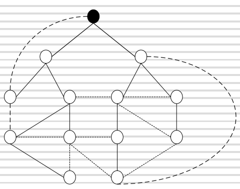
Introduction

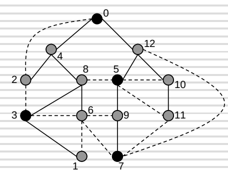
Alzoubi and Wan's algorithm



Construct a rooted spanning tree from the original network topology.

Introduction

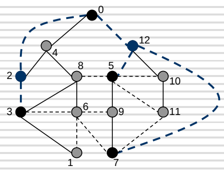
Alzoubi and Wan's algorithm



Color each node to be black or grey based on its rank (level, ID). The node with the lowest rank marks itself black. All the black nodes form an MIS.

Introduction

Alzoubi and Wan's algorithm



Connect the nodes in the MIS to form a CDS.

Performance Ratio = 8

Introduction

- PR = |CDS|/|opt|
- PR is determined by
  - How large an MIS is compared to a MCDS
  - How many vertices are required to connect an MIS
- PR = 8 (2002)

Introduction

- It has been proved that

$$\mathbf{MIS(G) \leq 4MCDS(G)+1}$$

- In this paper

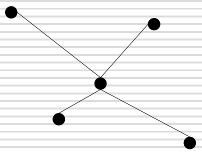
$$\mathbf{MIS(G) \leq 3.8MCDS(G)+1.2}$$

Preliminaries

- N(x): neighbor area of x, a unit disk centered at x
- N(G): the union of all neighbor areas of its vertices.
- u and v are adjacent if $|uv|<1$ and independent if $|uv|>1$

Preliminaries

Lemma 1 The neighbor area of a vertex contains at most five independent vertices.



$\angle x_{i-1} x x_i > 60^o$

$k \leq 5$

Preliminaries

Unit arc-triangle

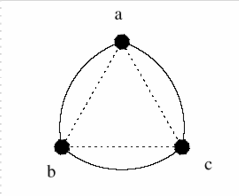


Figure 1: Unit arc-triangle *abc*.

Every two points in a Unit arc-triangle have distance at most 1.

u

v x1

Preliminaries

Lemma 2 A unit arc-triangle cannot contain two independent vertices.

Main Results

- Lemma 3 The neighbor area of two adjacent vertices contains at most 8 independent vertices.
- Lemma 4 For any unit disk graph, there exists a minimum spanning tree such that every vertex has degree at most five.
- Lemma 5 Every tree T with at least three vertices has a non-leaf vertex adjacent to at most one non-leaf vertex.

Main Results

Theorem For any unit disk graph *G*, the size of a maximal independent set is at most *3.8cds(G)+1.2* where *cds(G)* is the size of a minimum connected dominating set.

Main Results

Corollary

The best performance ratio of the current approximation algorithms for the MCDS is reduced from 8 to 7.8.

Theorem

- G: a CDS in the given UDG
- T: MST of G where every vertex has degree at most five.
- |T|: the number
- By induction on most 3.8|T|+1. vertices in the r

Lemma 4 For any unit disk graph, there exists a minimum spanning tree such that every vertex has degree at most five.

Theorem

- |T| = 1, |I
- |T| = 2, |MIS|≤ 3

Lemma 3 The neighbor area of two adjacent vertices contains at most 8 independent vertices.

of a vertex contains at most five independent vertices.

Theorem

$|T| \geq 3$

A non-lea ... to at most one ... v is adjacent ...

Lemma 5 Every tree T with at least three vertices has a non-leaf vertex adjacent to at most one non-leaf vertex.

T: MST of G where every vertex has degree at most five.

$x_1, \ldots, x_k$ $(k \leq 4)$: other neighbors of v

Theorem

- Each x_i's (1 ... contains at ... ertices which are a ...
- The neighbor a ... s at most 7 indep ... are also indepe ...

Lemma 3 The neighbor area of two adjacent vertices contains at most 8 independent vertices.

Lemma 1 The neighbor area of a vertex contains at most 5 independent vertices.

Theorem

The neighbor area of T-{v, $x_1, \ldots, x_k$} contains at most 3.8(|T|-k-1)+1.2=A independent vertices.

The neighbor area of T contains at most A+7+4(k-1) = 3.8|T|+1.2+0.2(k-4)$\leq$ 3.8|T|+1.2 independent vertices.

Discussion

Conjecture:

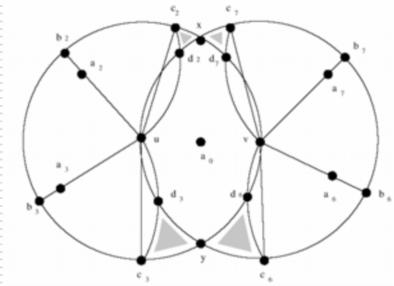
The neighbor area of a 4-star subgraph in a unit disk graph contains at most twenty independent vertices.

If this is true, Theorem 1 can be improved from 3.8 to 3.6.

Lemma 3

The neighbor area of two adjacent vertices contains at most 8 independent vertices.

Lemma 3



Lemma 3

- I = independent set contained in N(u)∪N(v), suppose 9≤|I|≤10 and A = N(u)∩N(v) contains k vertic… in I.

  |I|=(5…

- Let x, y be tw… boundaries o… yvx ≥ 120°.

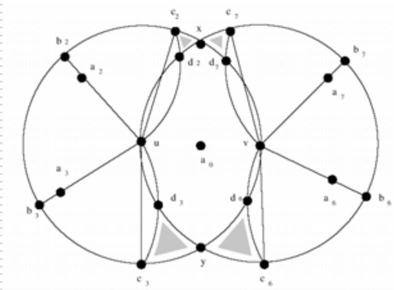
  |I|=…

A contains exactly … =9

Lemma 1 The neighbor area of a vertex contains at most five independent vertices.

$\angle x_{i-1}xx_i > 60^o$

Lemma 3



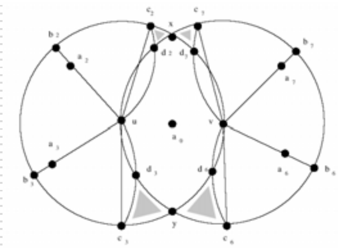
Lemma 3

- $a_0, a_1, \ldots, a_4$ lie counter-clockwisely in N(u)
- $a_0, a_5, \ldots, a_8$ lie counter-clockwisely in N(v)
- Let ub_i be the r… …, 4 and vb_i be th… $i=5, \ldots, 8$.
- Draw four unit … vb_6c_6, vb_7c_7.

Lemma 2 A unit arc-triangle cannot contain two independent vertices.

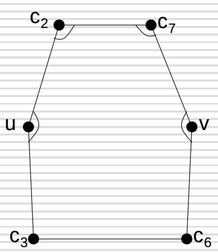
a_1, a_4, a_5, a_8 must lie in the four small dark areas.

Lemma 3



$\angle b_2ub_3 > 60^o$ $\quad \angle c_2ub_2 = \angle b_3uc_3 = 60^o$

Lemma 3



$\angle c_3uc_2 < 180^o$

$\angle c_7vc_6 < 180^o$

$\angle uc_2c_7 + \angle c_2c_7v > 180^o$

or

$\angle vc_6c_3 + \angle c_6c_3u > 180^o$

Lemma 3

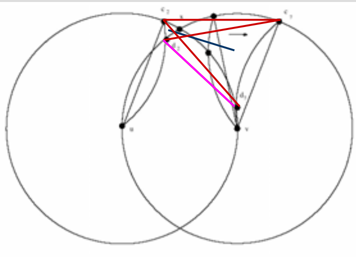


Figure 3: Turn unit arc-triangle vb_7c_7 until $vc_7 \parallel uc_2$.

c_2uvc_7 is a parallelogram

$|c_2c_7| = |uv| \leq 1$

The distance between two points in xc_2d_2 and xc_7d_7 cannot exceed $\max(|c_2c_7|, |c_2d_7|, |d_2c_7|, |d_2d_7|)$

$|d_2d_7| \leq \max(|c_2d_7|, |d_2c_7|) \leq 1$

Lemma 3

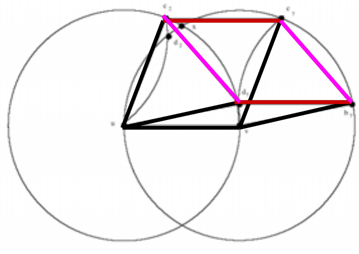


Figure 4: Move u until $|uv| = 1$.

$|uv| = |vb_7| = |b_7d_7| = |d_7u| = 1$

uvb_7d_7 and $c_2d_7b_7c_7$ are parallelograms.

$|c_2d_7| = |c_7b_7| = 1$

$|c_2d_7| \leq 1$
$|d_2c_7| \leq 1$

Lemma 4

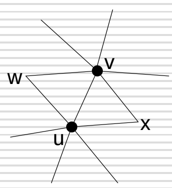
For any unit disk graph, there exists a minimum spanning tree such that every vertex has degree at most five.

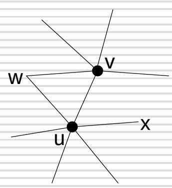
Lemma 4

T is a minimum spanning tree:

- Two edges meeting at a vertex form an angle of at least 60º.
  - Every vertex in *T* has degree at most 6.
- If two edges form an angle of exactly 60º, then they have the same length.

Lemma 4



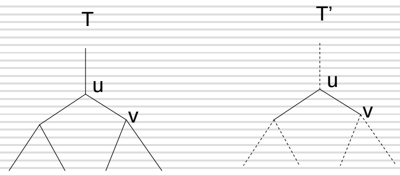


Lemma 5

Every tree T with at least three vertices has a non-leaf vertex adjacent to at most one non-leaf vertex.

37

Lemma 5



Every leaf of *T'* is a non-leaf vertex of *T* which is adjacent to at most one non-leaf vertex

38