Construction Algorithms for *k*-Connected *m*-Dominating Sets in Wireless Sensor Networks

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Outline

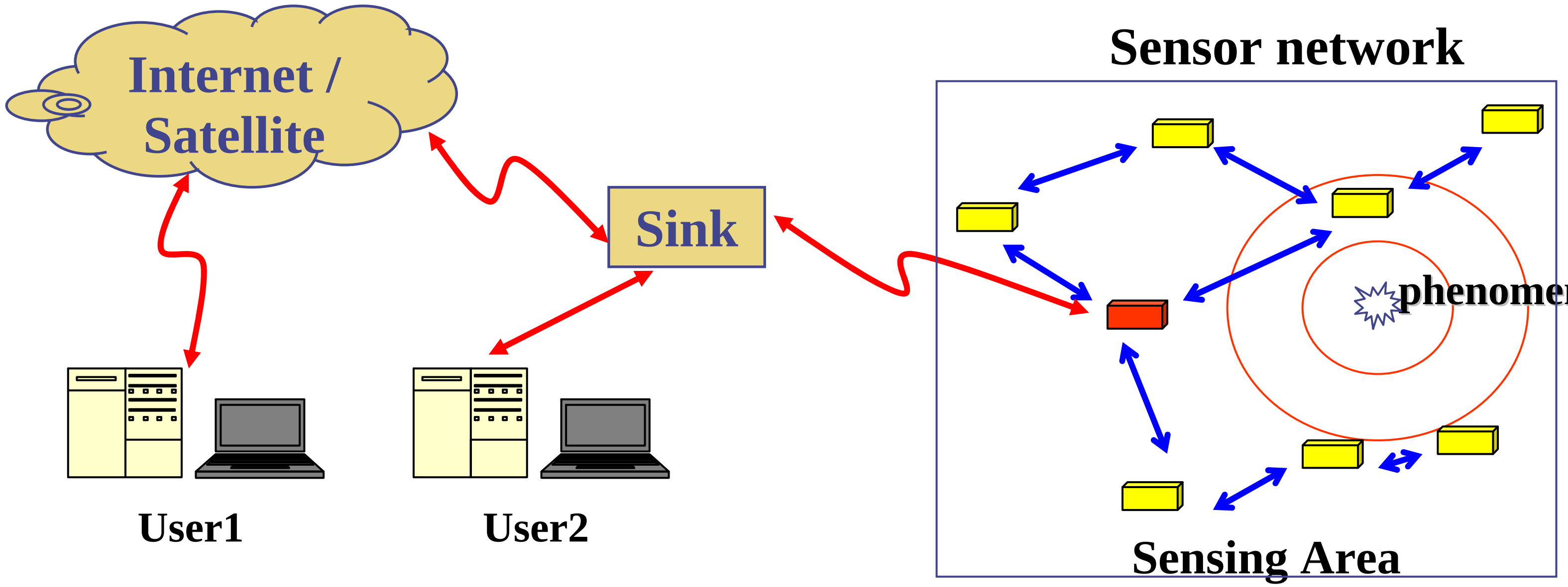
- Introduction
- Construction algorithms
  - MDSA: 1-connected m-dominating
  - LDA: distributed k-connected m-dominating
  - ICGA: centralized k-connected m-dominating
- Simulation results
- Conclusion

Introduction

- Wireless Sensor Networks
- Connected Dominating Sets
- Motivation

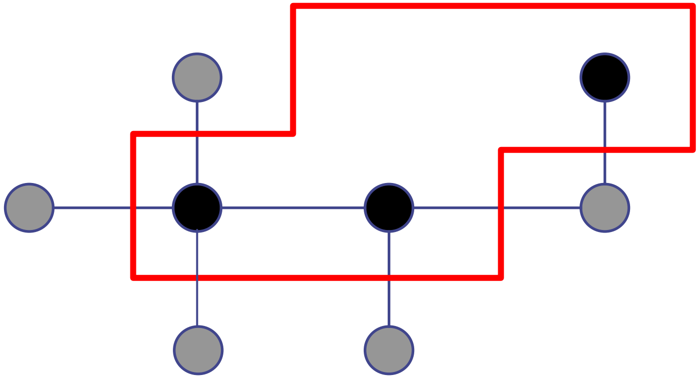
Wireless Sensor Networks

A Wireless Sensor Network (WSN) is an ad hoc **wireless network** which consists of a huge amount of static or mobile **sensors**. The sensors collaborate to sense, collect, and process the raw information of the **phenomenon** in the sensing area (in-network), and transmit the processed information to the **observers**.



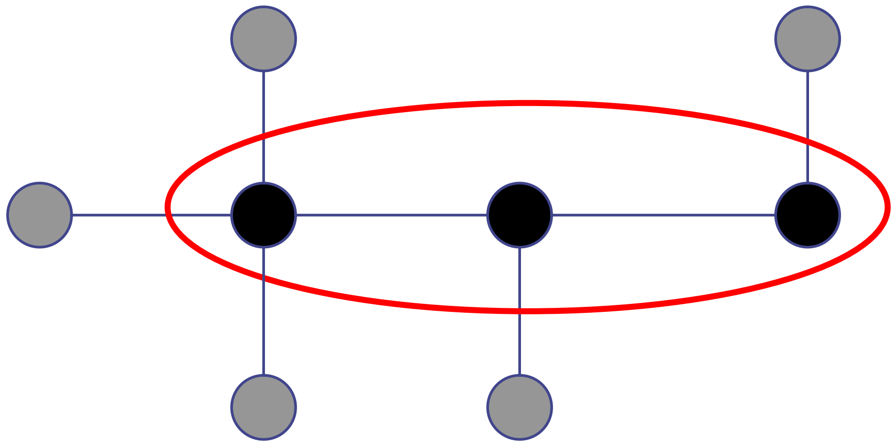
Connected Dominating Sets

A **dominating set** (DS) is a subset of all the nodes such that each node is either in the DS or adjacent to some node in the DS.



Connected Dominating Sets

A **connected dominating set** (CDS) is a subset of the nodes such that it forms a DS and all the nodes in the DS are connected.



Motivation

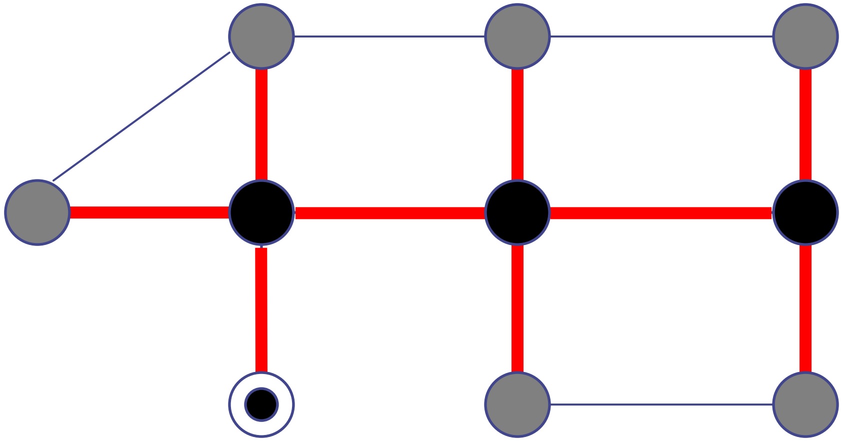
Flooding	CDS
Redundancy Contention Collision	Reduction of communication overhead
Unreliability	Reliability

CDS is used as a virtual backbone in wireless networks.

Motivation

Applications of CDS: Broadcast

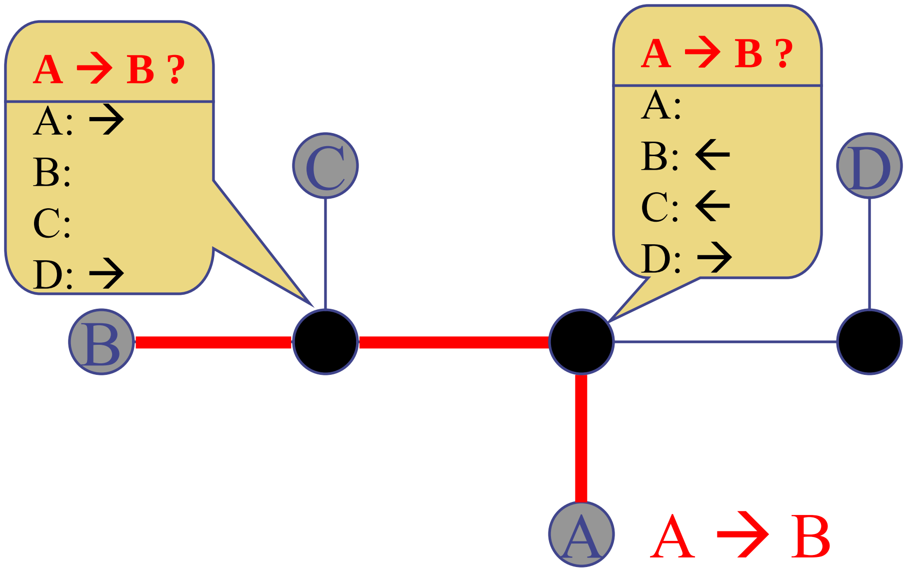
- Only nodes in CDS relay messages
- Reduce communication cost
- Reduce redundant traffic



Motivation

Applications of CDS: Unicast

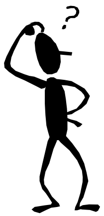
- Only nodes in CDS maintain routing tables
- Routing information localized
- Save storage space



Motivation

CDS plays an important role in WSNs.

New challenges



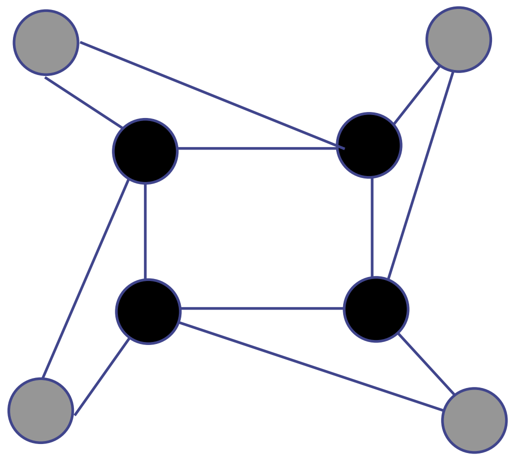
Does a CDS provide fault tolerance?

Does a CDS provide routing flexibility?

Motivation

k-Connected *m*-Dominating Set (*kmCDS*)

- *k*-connected
  – A graph is *k*-connected if and only if it contains *k* independent paths between any two nodes.
- *m*-dominating
  – If each node not in *C* is dominated by at least *m* nodes in *C*, then *C* is an m-dominating set.



A *kmCDS* provides fault tolerance and routing flexibility.

Motivation

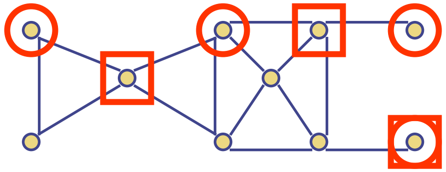
Related works

- [9] On Constructing *k*-Connected *k*-Dominating Set in Wireless Network, IEEE International Parallel & Distributed Processing Symposium, 2005 .
  - 3 K-Gossip-based algorithms: $k=m$
- [10] 2-connected Virtual Backbone in Wireless Network, Accepted by IEEE Transactions on Wireless Communications, 2007.
  - CDSA: $k=2$, $m=1$, 64-approximation centralized algorithm
- [11] Algorithms for Minimum m-Connected k-Dominating Set Problem, COCOA 2007, LNCS 4616, pp.182-190, 2007.
  - 3 centralized algorithms: $k=1$; $k=2$; $3 \leqslant k \leqslant m$
- [12] On approximation algorithms of *k*-connected *m*-dominating sets in disk graphs, Accepted by Theoretical Computer Science, 2007.
  - 1 centralized algorithm: input graph at least $max(k, m)$-connected; not easy to implement
- [13] Constructing *k*-Connected *m*-Dominating Sets in Wireless Sensor Networks, Military Communications Conference, Orlando, FL, October 29-31, 2007.
  - Centralized CGA: probabilistic algorithm
  - Distributed DDA: high message complexity

Construction Algorithms

- Preliminaries
- MDSA
- LDA
- ICGA

Preliminaries



Maximal Independent Set (MIS) is a maximal set of pair-wise non-adjacent nodes.

MIS ⟷ DS

Preliminaries

- Open neighbor set of v: $N(v) = \{u \mid (v, u) \in E\}$
- Closed neighbor set of v: $N[v] = N(v) \cup \{v\}$
- Common node set of u and v: $S_{cn}(u, v) = N[v] \cap N[u]$.
- Common black node set of u and v: $S_{cbn}(u, v)$ = {All the black nodes in $S_{cn}(u, v)$}.
- v's local graph: $GL(v)$ is the graph induced by v and all of its neighbors.
- Local vertex connectivity of node v: $LVC(v)$ = vertex connectivity degree of $GL(v)$.

MDSA

General idea:

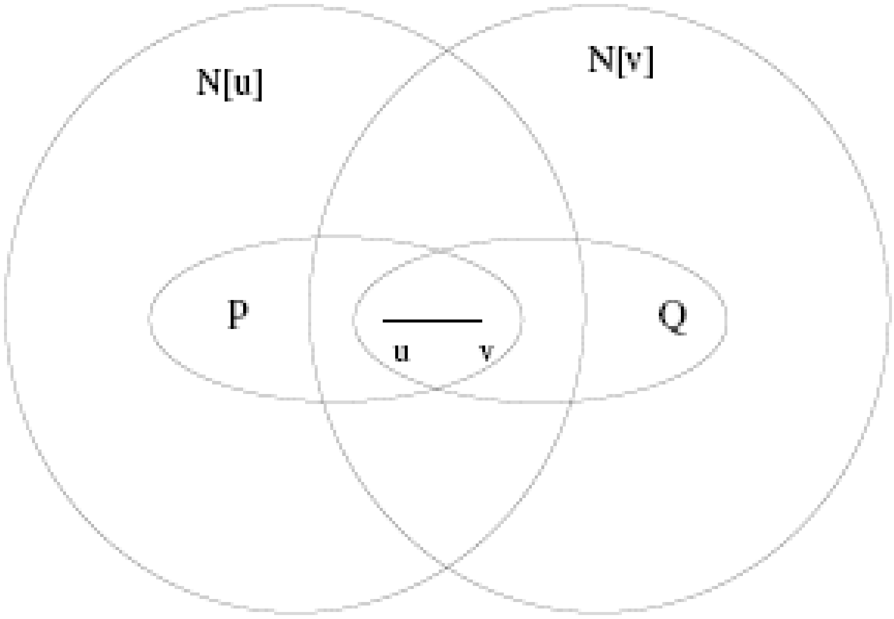
1. Construct a $(1, 1)$-CDS $C_{11} = M_1 \cup C_0$, where M_1 is an MIS and C_0 is the set of connectors connecting the nodes in M_1.
2. Construct a $(1, m)$-CDS $C_{1m} = C_{11} \cup M_2 \cdots \cup M_m$ where M_i is an MIS from $G \backslash (C_{11} \cup M_2 \cdots \cup M_{i-1})$.

Outcome:

- The performance ratio of MDSA is $\boldsymbol{(5+5/m)}$ for $m \leqslant 5$ and 7 for $m \geqslant 6$.
- The time complexity of MDSA is $\boldsymbol{O(m \cdot Diam)}$ and the message complexity is $\boldsymbol{O(m(\Delta+1)|V|)}$, where Δ is the maximum node degree and *Diam* is the network diameter.

LDA

Fact



If $|P \cap Q| \geqslant k$ and $LVC(P) = LVC(Q) = k$,
then $LVC(P \cup Q) = k$.

LDA

General idea:

1. Construct a *(1, m)*-CDS C_{1m} using MDSA.
2. Negotiate a common black node set: Every black node in C_{11} negotiates with its parents or siblings who are also in C_{11} about the S_{cbn} to make $|S_{cbn}| \geqslant k$. (Unnecessary if $k = 2$)
3. Build a local *k*-connected subgraph (*Algorithm 1*): Every black node in C_{11} builds a local *k*-connected graph G_k which includes all the black neighbors in C_{1m} and S_{cbn}, and marks all the nodes in G_k black.

LDA

Outcome:

- The performance ratio of LDA is $\boldsymbol{max\{5/m, 1\}2\Delta}$, where Δ is the maximum node degree.

- The message complexity of LDA is $\boldsymbol{O(\Delta|V|)}$ and time complexity is $\boldsymbol{O((m + \Delta) \cdot Diam)}$, where Δ is the maximum node degree and *Diam* is the network diameter.

ICGA

Facts

- If G is a k-connected graph, and G' is obtained from G by adding a new node x with at least k neighbors in G, then G' is also a k-connected graph.
- Given a k-connected graph G and a connected set F which can k-dominate G, the graph G' composed by $G \cup F$ is $(k + 1)$-connected.

General idea (*Algorithm 2*):

1. Construct a $(1,m)$-CDS C_{1m} using MDSA.
2. Sequentially augment set C_{1m} for k-connectivity to obtain a (k,m)-CDS.

ICGA

Outcome:

- The performance ratio of ICGA is f, where

$$
f = \begin{cases}
\begin{cases} 5k + \frac{5}{m} + 5H_{k-1} & m \le 5 \\ 7k & m \ge 6 \end{cases} & k \le 6 \\
\\
\begin{cases} 7k - 7 & m \le 5 \\ 7k & m \ge 6 \end{cases} & k \ge 7
\end{cases}
$$

and H_{k-1} is the $(k-1)th$ harmonic number.

and H_{k-1} is the $(k-1)^{th}$ harmonic number.

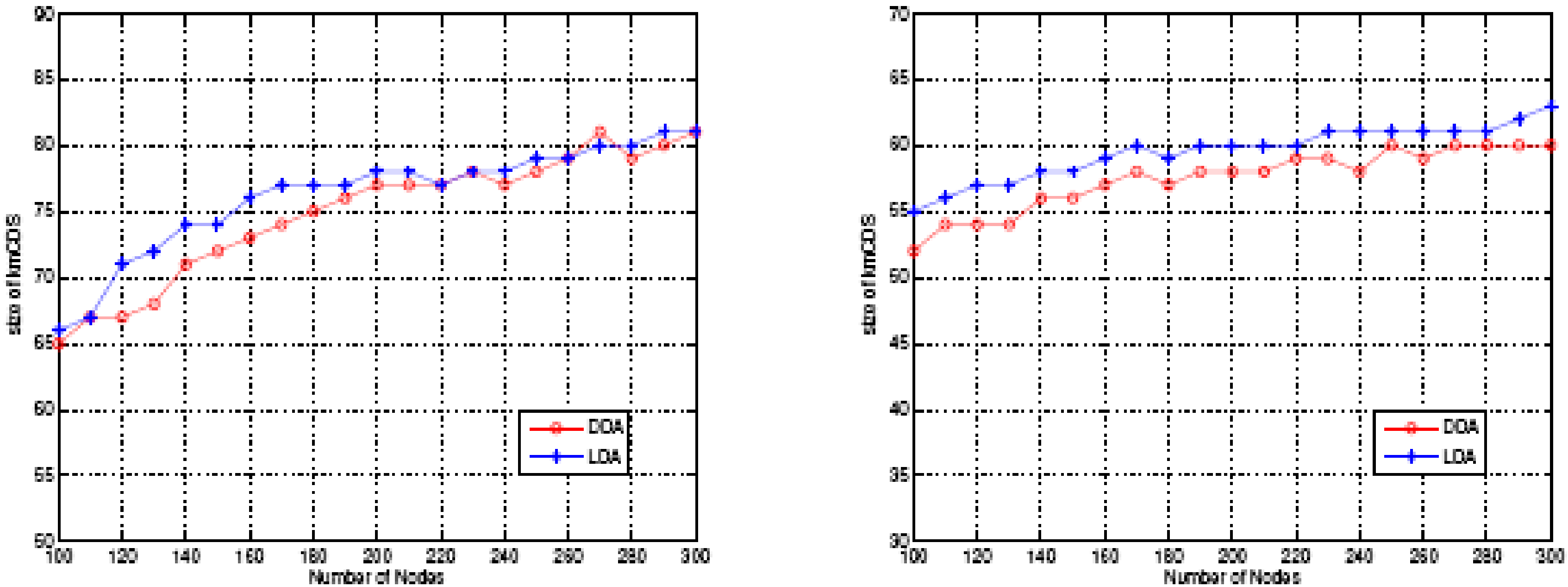
- The time complexity of ICGA is $O(|V|^{3.5}|E|)$.

Performance ratio of DDA

DDA: Y. Wu, F. Wang, M. T. Thai, and Y. Li, Constructing *k*-Connected *m*-Dominating Sets in Wireless Sensor Networks, MILCOM 2007, Orlando, FL, October 29-31, 2007.

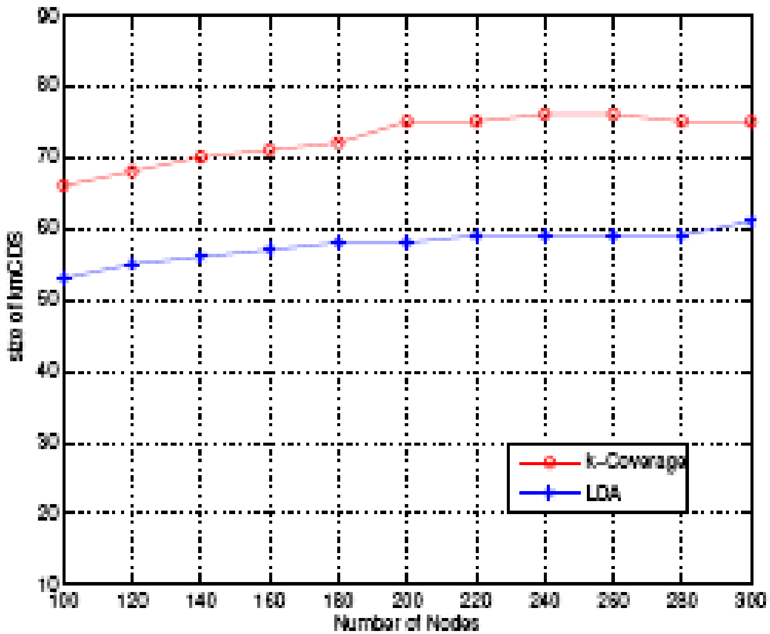
- The performance ratio of DDA is $\boldsymbol{(5+5/m)(k^2+1)}$ for $m \leqslant 5$ and $7\boldsymbol{(k^2+1)}$ for $m \geqslant 6$, if we use MDSA in phase 1 to construct a $(1,m)$-CDS C_{1m}.

Simulation Results

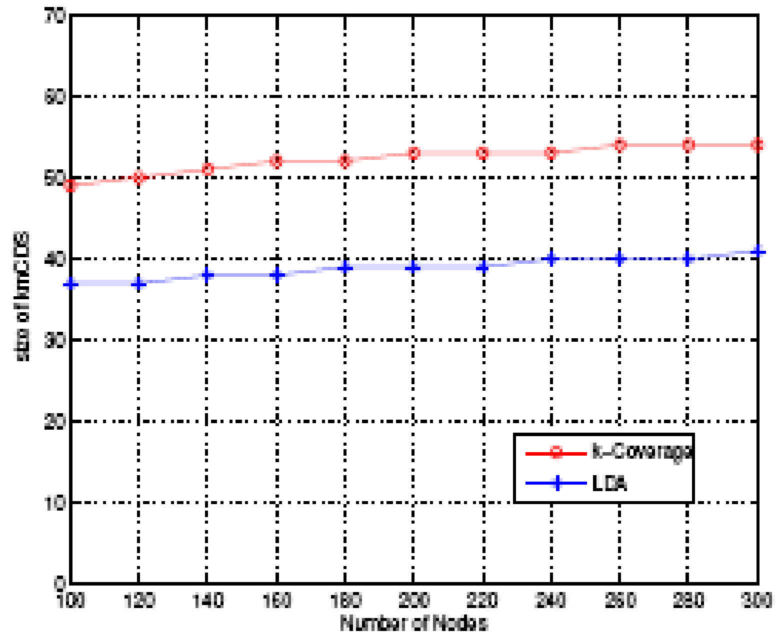


(a) $k = 4, m = 4$ (b) $k = 3, m = 4$

	DDA [13]	LDA
Size of *km*CDS	The size of the *km*CDS obtained by LDA is 5% larger than that by DDA.	
Message complexity	$O(\Delta^2\|V\|)$	$O(\Delta\|V\|)$



(c) $k = 3, m = 3$



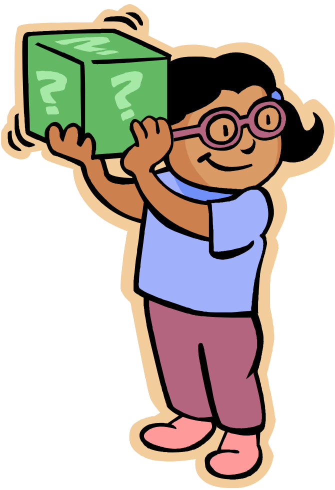
(d) $k = 2, m = 2$

	k-coverage [9]	LDA
k and *m*	*k* = *m*	General *k* and *m*
Size of *km*CDS	The size of the *km*CDS obtained by LDA is 25.5% and 20.5% smaller for *k*=*m*=*2* and *k*=*m*=*3* respectively than that by *k*-Coverage.	

Conclusion

- A *km*CDS provides fault tolerance and routing flexibility.
- MDSA: a distributed *(1,m)*-CDS construction algorithm
- LDA: a distributed *km*CDS construction algorithm for general *k* and *m* with low message complexity.
- ICGA: a centralized algorithm with a constant performance ratio and it guarantees obtaining a *km*CDS.
- Derive a tighter bound of the performance ratio of DDA [13].

Thanks



Q & A