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# Localized Construction of Connected Dominating Set in Wireless Networks

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# Algorithm Classification

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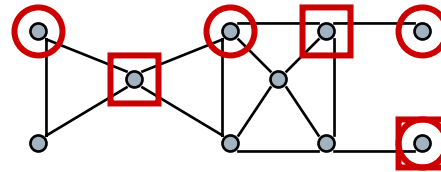
- ❑ Distributed VS. Centralized
- ❑ Completely localized VS. Serialized

	<i>PR</i>	<i>Stage</i>	<i>Connectivity Information</i>
<i>Wu's</i>	O(n)	1-stage	Considered
<i>Alzoubi's</i>	192	2-stage	Not-considered
<i>r-CDS</i>	<b>172</b>	<b>1-stage</b>	<b>Considered</b>

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# Key idea

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**Maximal Independent Set (MIS)** is a maximal set of pair-wise non-adjacent nodes.

**MIS**  $\longleftrightarrow$  **DS**

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# Notations

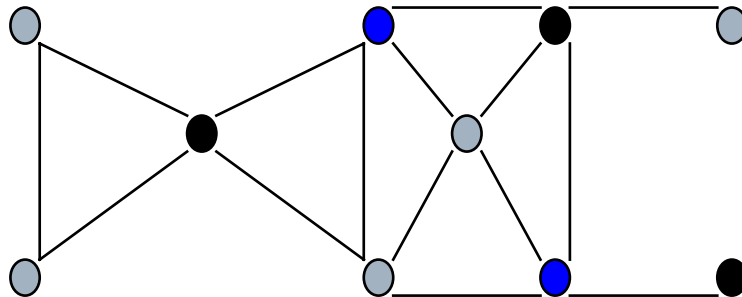
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Black nodes: MIS nodes.

Blue nodes: nodes connecting black nodes.

Grey nodes: dominatee nodes.

Black nodes + Blue nodes = dominating nodes (CDS)

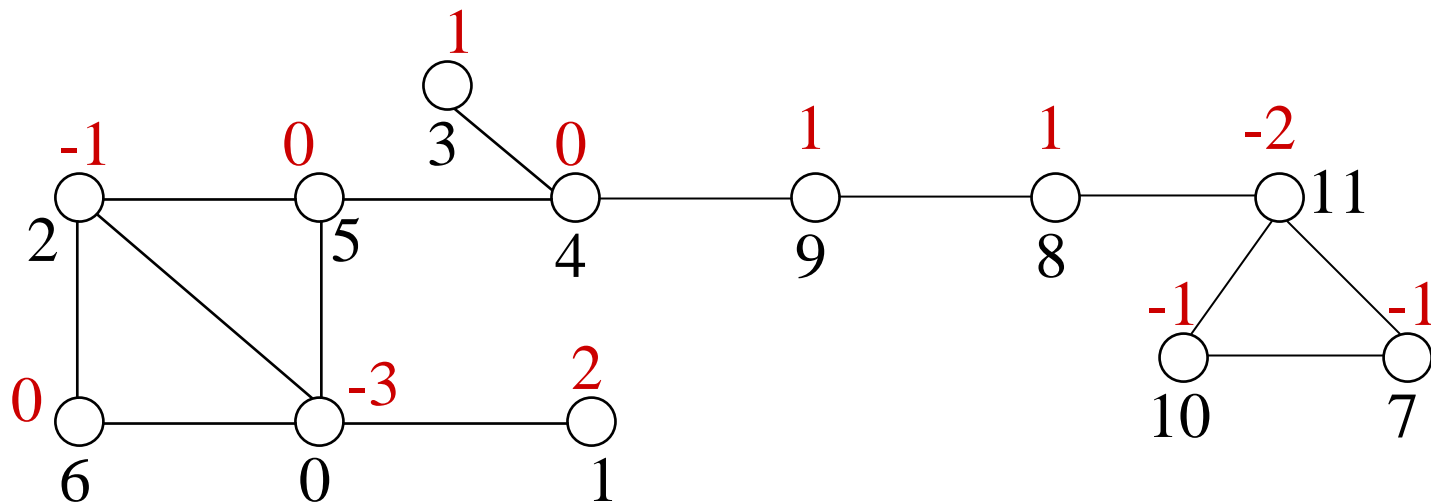


# r-CDS

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For each node  $u$

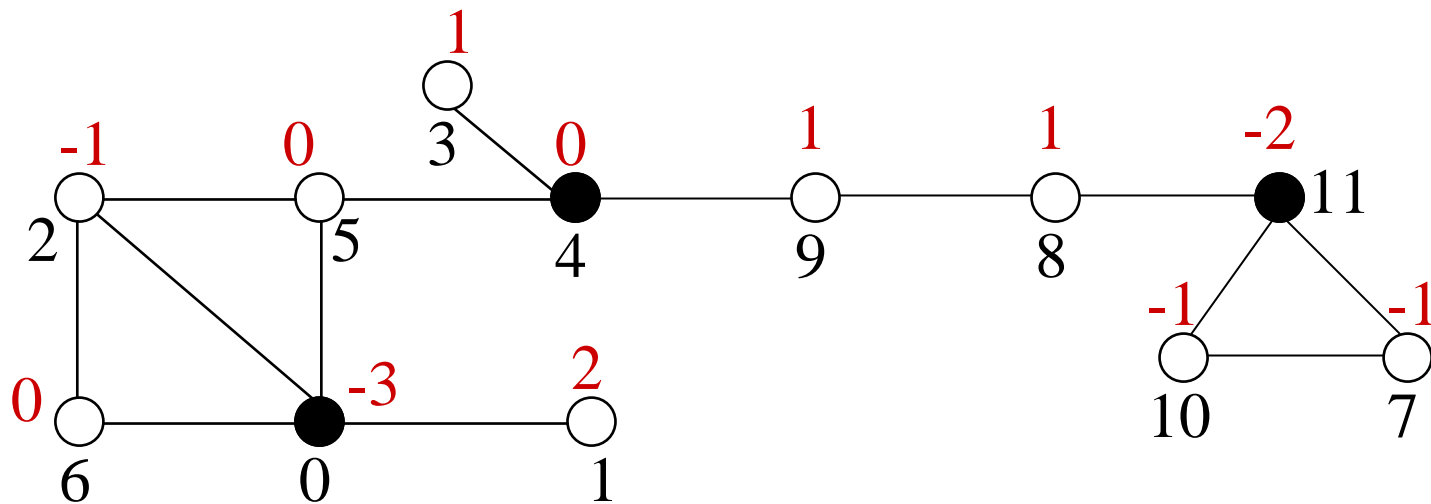
$r(u)$  = the number of 2-hop-away neighbors  $- d(u)$   
where  $d(u)$  is the degree of node  $u$



# r-CDS (Cont.)

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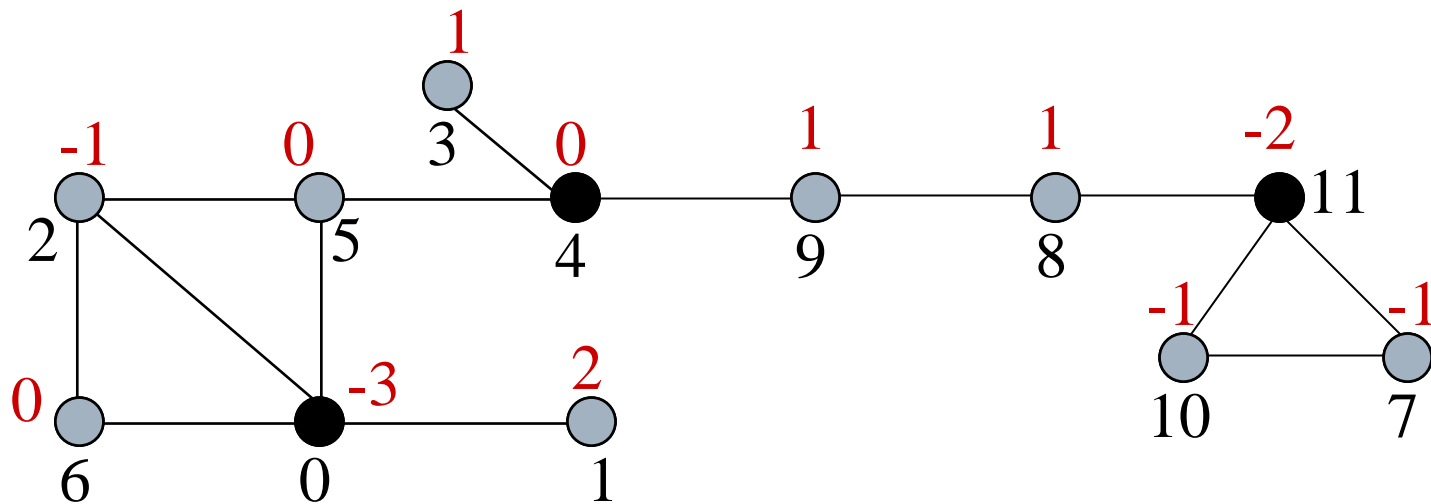
Node  $u$  with the smallest  $\langle r, \text{deg}, \text{id} \rangle$  within its neighborhood becomes black and broadcast a BLACK message where  $\text{deg}$  is the **effective degree**.



# r-CDS (Cont.)

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If  $v$  receives a BLACK message from  $u$ ,  $v$  becomes grey and broadcasts a GREY message containing  $(v, u)$ .

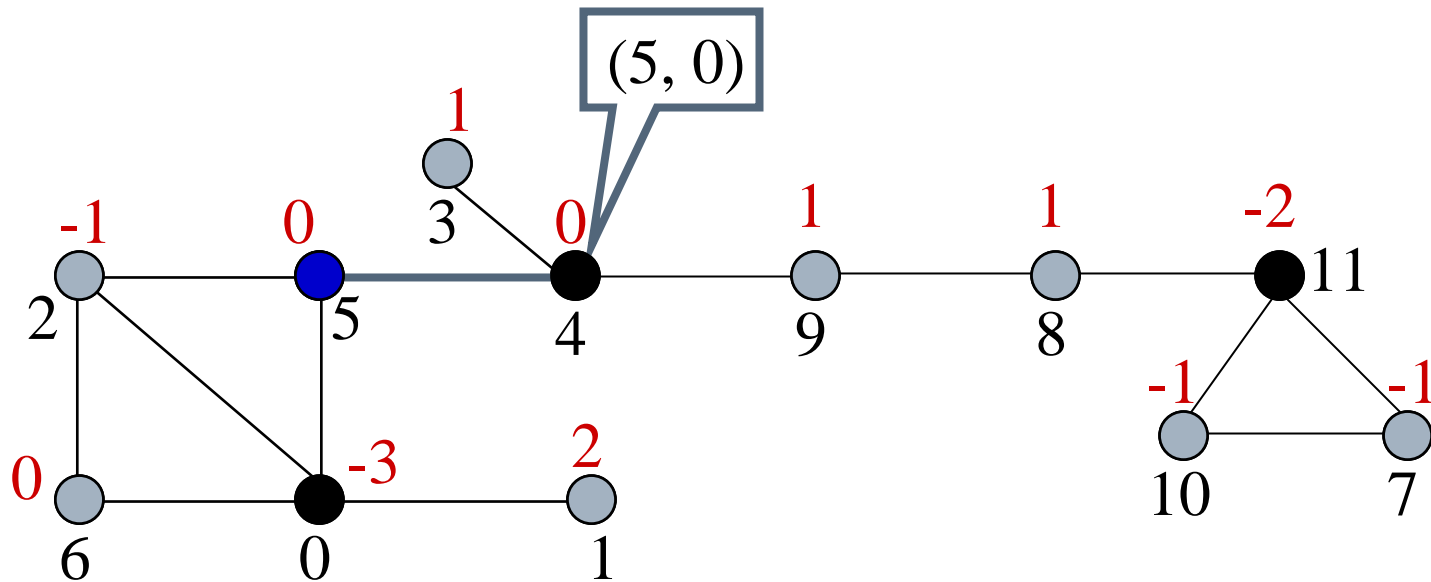


# r-CDS (Cont.)

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- ❑ black node  $w$  receives a GREY message  $(v, u)$
- ❑  $w$  not connected to  $u$

*Color  $v$  blue*

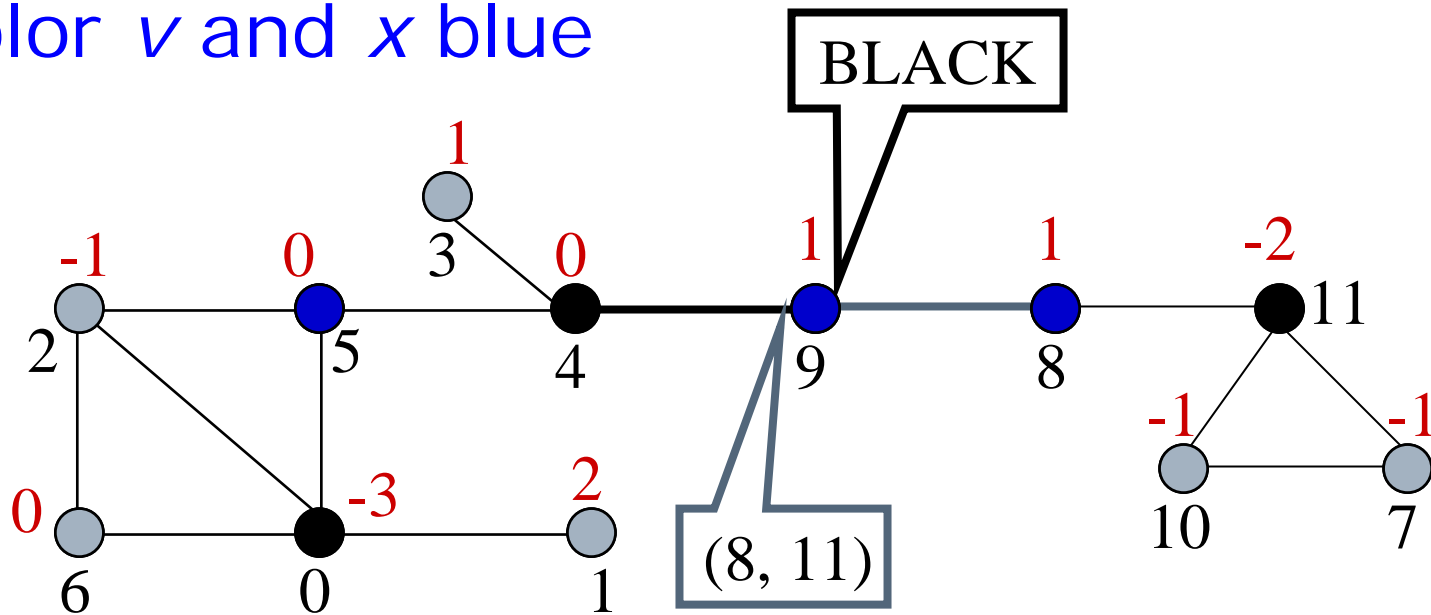


# r-CDS (Cont.)

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- ❑  $v$  has received a GREY message  $(x, y)$
- ❑  $v$  receives a BLACK message from  $u$
- ❑  $y$  &  $u$  not connected

color  $v$  and  $x$  blue



# Analysis

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*Lemma 1* All the black nodes form an MIS.

- No 2 adjacent white nodes become black simultaneously.
  - All the neighbors of a black node become grey.
  - Black nodes form an independent set.
  - Grey nodes can no longer become black.
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# Analysis (Cont)

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*Lemma 2* Let  $d$  denote the number of hops between any pair of black nodes, then  $d=2$  or  $d=3$ .

- Black nodes form an MIS.
  - A grey node must have a black neighbor.
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# Analysis (Cont)

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*Theorem 1* All the black nodes and blue nodes form a CDS.

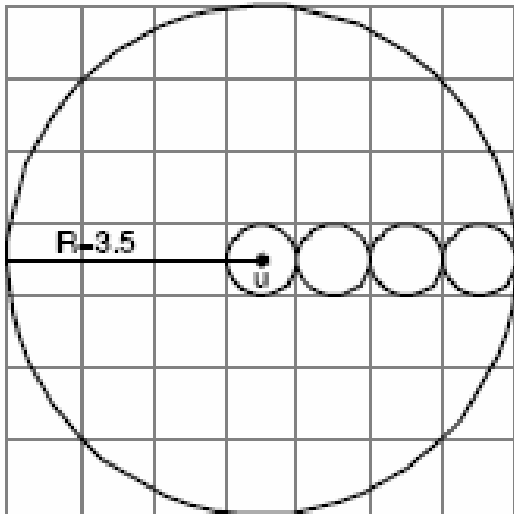
*Theorem 2* The time complexity is  $O(\Delta)$  where  $\Delta$  is the maximum node degree and the message complexity is  $O(n \Delta^2)$ .

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# Analysis (Cont)

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*Lemma 3:* Let  $S$  be any MIS of a UDG  $G$ . For any node  $u$  in  $S$ , the number of the nodes in  $S$  that are at most three hops away from  $u$  is at most 42.



$$D = \frac{nd^2\pi}{4A(Q)}$$

$$D \leq \frac{n}{\left[1 - \frac{\sqrt{3}}{2} + \sqrt{\frac{3}{4} + \frac{2\sqrt{3}}{\pi}(n-1)}\right]^2}$$

where  $D$  is a maximum density of packing  $n$  equal circles in another larger circle.

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# Analysis (Cont)

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*Theorem 3* The performance ratio of  $r$ -CDS is 172.

- The size of any MIS  $S$  in a graph is at most  $4opt + 1$  where  $opt$  is the size of any optimal CDS of the graph.
  - $|C| \leq 2 \cdot 42 |S| / 2 + |S| \leq 172opt + 43$
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