Construction Algorithms for k-Connected m-Dominating Sets in Wireless Sensor Networks

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ABSTRACT

A Connected Dominating Set (CDS) working as a virtual backbone is an effective way to decrease the overhead of routing in a wireless sensor network. Furthermore, a k-Connected m-Dominating Set (kmCDS) is necessary for fault tolerance and routing flexibility. Some approximation algorithms have been proposed to construct a kmCDS. However, most of them only consider some special cases where $k = 1, 2$ or $k \leq m$, or are not easy to implement, or have high message complexity. In this paper, we propose a novel distributed algorithm LDA with low message complexity to construct a kmCDS for general k and m whose size is guaranteed to be within a small constant factor of the optimal solution when the maximum node degree is a constant. We also propose one centralized algorithm ICGA with a constant performance ratio to construct a kmCDS. Theoretical analysis as well as simulation results are shown to evaluate the proposed algorithms.

Categories and Subject Descriptors

C.2.1 [**Network Architecture and Design**]: Network topology, Wireless communication; G.2.2 [**Graph Theory**]: Network problems, Graph algorithms

General Terms

Algorithms, Design, Performance, Theory

Keywords

Connected Dominating Set, k-Connected m-Dominating Set, Fault-tolerance, Wireless Sensor Networks, Localized algorithm

1. INTRODUCTION

Wireless Sensor Networks (WSNs) are now being used in many applications, such as environment and habitat monitoring, traffic control, and *etc.* Since there is no fixed or predefined infrastructure in WSNs, the simplest routing method is flooding, which not only wastes energy of nodes but also diminishes the throughput of the whole network. Therefore, a Connected Dominating Set (CDS) has been recommended to serve as a virtual backbone for a WSN to reduce routing overhead. The nodes in a CDS are called *dominators*, the others are called *dominatees*. Having such a CDS simplifies routing by restricting the main routing tasks to the dominators only. Unlike in a traditional way where a node broadcasts the packet whenever it first receives one, all dominatees only need to send packets to their closest dominator(s) and only the dominators forward the packets towards their destinations.

Fault tolerance and routing flexibility are necessary for routing since nodes in WSNs are prone to failures and nodes may have mobility and turn on and off frequently. Thus, it is important to maintain a certain degree of redundancy in a CDS. Unfortunately, a CDS only preserves 1-connectivity and it is therefore very vulnerable. In this paper, we investigate how to construct a k-Connected m-Dominating Set (kmCDS). The requirement of k-connectivity guarantees that between any pair of dominators there exist at least k different paths. The requirement of m-domination takes care of fault tolerance and robustness for dominatees, which ensures that every dominatee has at least m adjacent dominator neighbors.

Several approximation algorithms for constructing kmCDSs have been proposed in the literature. However, all of these algorithms only consider some special cases for $k = 1, 2$ or $k \leq m$, or are not easy to implement, or have high message complexity. The main contribution of our work is that we propose a distributed kmCDS construction algorithm for general k and m with low message complexity. And we also propose a centralized algorithm ICGA which has a constant performance ratio, and it also improves the best kmCDS construction algorithm CGA [13] which cannot guarantee obtaining a kmCDS unless the whole graph is k-connected. We also show the performance ratio of ICGA which CGA can not provide as well. Furthermore, we specify the tight bound of the performance ratio for another distributed kmCDS construction algorithm DDA [13].

The remainder of this paper is organized as follows. In Section 2, we review some existing kmCDS construction algorithms. In Section 3, we present some preliminaries and our network model that are necessary for illustrating our algorithms. In section 4, we show an algorithm MDSA to construct a 1-connected m-dominating set, based on which, our algorithm LDA and ICGA are designed. Algorithm LDA is presented in Section 5. A centralized algorithm, ICGA, is

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proposed in Section 6. We also analyze the DDA algorithm [13] in Section 7 to give a tight bound of its performance ratio. In Section 8, we evaluate the performances of our proposed algorithms through simulations. Finally, we conclude this paper and discuss some future research directions in Section 9.

2. RELATED WORK

Efficient distributed algorithms for constructing CDSs in WSNs were well studied in [1, 2, 3, 4, 5, 6, 7, 8]. Wu *et al.* [1] proposed their localized connected dominating set method using a marking process where a node is marked true if it has two unconnected neighbors. It is shown that the set of marked nodes forms a CDS. In [4], Dai *et al.* further extended the pruning rules to k-hop neighborhoods in order to achieve better results. Alzoubi *et al.* [3, 6] proposed a approximation method to construct a minimum CDS with performance ratio of 8. In [5], Chen *et al.* also proposed a localized algorithm to build a CDS for topology maintenance, where a node becomes a dominator when two of its neighbors cannot reach each other either directly or via one or two dominators. In [7], a distributed algorithm r-CDS was proposed whose performance ratio is 172. r-CDS is a completely localized one-phase algorithm where each node only needs to know the connectivity information within its 2-hop-away neighborhood. In [8], another localized algorithm was proposed whose performance ratio is 147. This algorithm contains three steps. Step 1 constructs a forest in which each tree is rooted at a node with the minimum ID among its 1-hop-away neighbors. Step 2 collects neighborhood information, which is used in Step 3 to connect neighboring trees.

Although CDS construction has been investigated extensively, not too much research for kmCDS construction has been conducted in the literature. There exist several algorithms including centralized and distributed ones to construct a kmCDS. Most of them only consider some special cases where $k = 1, 2$ or $k \leq m$, or are not easy to implement, or have high message complexity. The most related works to us are [9], [10], [11], [12] and [13].

In [9], three localized K-CDS construction protocols were proposed. The first one is a probabilistic approach which is based on K-Gossip. In K-Gossip algorithm, each node decides its color with a probability based on the network size, transmission range, the value of k, and *etc.* The second is a deterministic approach which is an extension from the K-coverage condition. The last one is Color-Based K-CDS Construction. In Color-Based algorithm, each node randomly selects one of the k colors. Therefore, the whole network is divided into k-disjoint subsets based on colors. The major difference between our work and theirs is that their approaches only consider to construct a kmCDS where $k = m$, while our works constructs a kmCDS for general k and m.

In [10], the 64-approximation centralized algorithm Connecting Dominating Set Augmentation (CDSA) to construct a 2-connected virtual backbone was proposed. This algorithm first constructs a CDS, and then computes all the blocks and adds intermediate nodes to make all the backbone nodes being in the same block. Similarly, this work is also only for the case where $k = 2$ and $m = 1$.

In [11], three centralized algorithms were proposed. One is for constructing a 1-connected m-dominating set. Another is for constructing a 2-connected m-dominating set whose basic idea is similar to the work in [10]. The last one is for $3 \leq k \leq m$ which first constructs a k-connected k-dominating set and then sequentially constructs an MIS $m - k$ times.

In [12], a centralized algorithm was proposed which requires the input graph to be at least $\max(k, m)$ connected. First, a 1-m-CDS C_{1m} is built. Then C_{1m} is augmented to become a k-m-CDS sequentially. However, this algorithm is not easy to implement due to the difficulty in finding all the k-blocks or k-leaves from a graph.

In [13], one centralized algorithm CGA and one distributed algorithm DDA were proposed. The main idea of CGA is to construct a m-dominating set first and then augment this set to be k-connected by adding enough number of connectors. Although CGA can be implemented easily, it cannot guarantee obtaining a kmCDS. The main idea of DDA is the same as that of CGA except that DDA builds a 1-connected m-dominating set first. However, a lot of control messages are needed in DDA, which makes the message complexity of DDA very high.

In this paper, we first present a distributed kmCDS construction algorithm, LDA, for general k and m. LDA is a totally distributed algorithm which is preferred by WSNs, especially for large WSNs. It also has lower message complexity than others. For small networks, centralized algorithms are more suitable since they may have better results and may save communication cost compared with distributed algorithms. Therefore, we also propose a centralized algorithm ICGA which is better than CGA since CGA cannot always guarantee obtaining a kmCDS. We also show the performance ratio of ICGA which CGA can not provide.

3. PRELIMINARIES AND NETWORK MODEL

In this section, we present some definitions and notations that will be used later. In this paper, we are mainly interested in static symmetric multi-hop WSNs. The topology of a network is represented as a Unit Disk Graph (UDG), denoted as $G(V, E)$, where V is the node set and E is the edge set. The main purpose for us to choose a UDG as a network model is to illustrate the performance ratio of our algorithms. Otherwise, our network model can be relaxed to general undirected graphs. We also use nodes and vertices interchangeably in the context of graph theory and WSNs.

We use $N(v) = \{u|(v, u) \in E\}$ to represent the *open neighbor set* of vertex v. $N[v] = N(v) \cup \{v\}$ is called the *closed neighbor set* of v. Node v and all of its neighbors compose of node v's *local graph* denoted as $GL(v)$. As shown in Figure 1, node 1 and its neighbors compose of node 1's local graph. *Local vertex connectivity* of node v, denoted as $LVC(v)$, is the vertex connectivity degree of $GL(v)$. In Figure 1, $LVC(1) = 2$. A *common node set* of nodes v and u, denoted as $S_{cn}(v, u)$, is the set of $N[v] \cap N[u]$. That is $S_{cn}(v, u) = N[v] \cap N[u]$.

A subset of vertices of a graph G is called an *independent set* if there is no edge between any pair of vertices in this subset. It is a *maximal independent set* (MIS) if no more vertices can be added to it to preserve the independence property. A *Dominating Set* D of G is defined as a subset of V such that each node in $V \setminus D$ is adjacent to at least one node in D. Therefore, any MIS is a dominating set. A

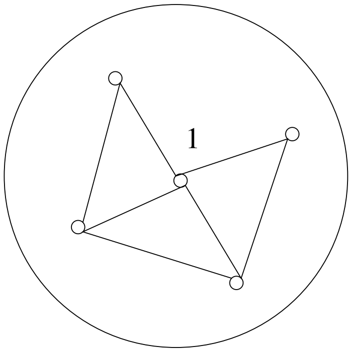


Figure 1: Node 1 and its neighbors compose of node 1's local graph.

Connected Dominating Set (CDS) C of G is a dominating set of G which induces a connected subgraph of G. The nodes in C are called *dominators*, otherwise *dominatees*. A graph G is said to be k *vertex connected* or k *connected* if for each pair of vertices there exists at least k mutually independent paths connecting them. In other words, the graph G is still connected even after the removal of any $k-1$ vertices from G. Given $v, u \in V$, a set $X \subseteq V - \{v, u\}$ is called a v, u-*separator* or v, u-cut if there is no path between v and u in $G \setminus X$. Clearly, the minimum size of an v, u-separator equals the maximum number of pairwise internally disjoint v, u-paths. Therefore, if the minimum size of any pair separators is larger than k, the whole graph G is at least k connected. A vertex is said to dominate itself and all of its neighbors. An m-*dominating set* D_m is a set $D_m \subset V$ such that every vertex in $V \setminus D_m$ is dominated by at least m vertices in D_m. A set $C \subset V$ is a k-*connected* m-*dominating set* (kmCDS) of graph $G(V, E)$ if the induced subgraph $G'(C, E')$ is k-vertex connected and the set C is also an m-dominating set of G.

Given two (N_i, ID_i), (N_j, ID_j) 2-tuple variables, where N_i is the degree of node i and N_j is the degree of node j, the *weight function* $W : (N, ID) \mapsto R$ satisfies:

$$\begin{aligned} & W(N_i, ID_i) > W(N_j, ID_j) \\ \Leftrightarrow \quad & N_i > N_j \\ & \text{or } (N_i = N_j \text{ \&\& } ID_i > ID_j). \end{aligned}$$

4. CONSTRUCTING 1-CONNECTED M-DOMINATING SET

Firstly, in this section we present an algorithm MDSA to construct a 1-connected m-dominating set which is based on a distributed CDS construction algorithm CDS-BD-D [14]. We choose CDS-BD-D because this algorithm is completely distributed, can be easily implemented and can obtain a CDS with a small size. MDSA is a fundamental component of our kmCDS construction algorithms.

Now we briefly introduce the basic idea of CDS-BD-D. The root first builds a BFS tree. After that, the root marks itself black and broadcasts a BLACK message. After receiving either BLACK or WHITE messages from all parents, a node u who has the highest $W(N, ID)$ among its sibling nodes marks itself white if it receives at least one BLACK message from its parents or siblings. Otherwise, it sends a CONNECT message to its parent who has the highest $W(N, ID)$. Both of the parent node and u mark themselves black and broadcast BLACK messages. After all nodes decide their colors, all black nodes form a CDS. The performance ratio of this algorithm is 6.096. We can modify this algorithm to construct an MIS by cancelling all the CONNECT messages.

Our MDSA algorithm consists of two phases:

1. Construct a CDS $C_{11} = M_1 \cup C_0$ using the CDS-BD-D algorithm, where M_1 is an MIS and C_0 is the set of connectors connecting the nodes in M_1.
2. Construct a 1-connected m-dominating set $C_{1m} = C_{11} \cup M_2 \cdots \cup M_m$ where M_i is an MIS from $G \backslash (C_{11} \cup M_2 \cdots \cup M_{i-1})$ constructed by using CDS-BD-D.

LEMMA 1. *Let $G = (V, E)$ be a UDG and m a constant such that $\delta_G \geq m - 1$, where δ_G is the minimum degree of G. Let D_m^* be a minimum m-dominating set of G and M an MIS of G. Then $|M| \leq \max\{\frac{5}{m}, 1\}|D_m^*|$.*

LEMMA 2. *$| M_1 \cup M_2 \cdots \cup M_m | \leq 5|D_m^*|$ for $m \leq 5$ and $| M_1 \cup M_2 \cdots \cup M_m | \leq 6|D_m^*|$ for $m \geq 6$, where M_i is an MIS from $G \backslash (M_1 \cup M_2 \cdots \cup M_{i-1})$.*

Lemma 1 and 2 were proved in [11].

THEOREM 1. *The set C_{1m} derived from MDSA is a 1-connected m-dominating set.*

PROOF. For each node $u \in G \setminus C_{1m}$, u is dominated by at least m different vertices in $M_1 \cup M_2 \cup \cdots \cup M_m$. Since a CDS is constructed first, then $C_{11} = M_1 \cup C_0$ is connected. Thus, $C_{1m} = C_{11} \cup M_2 \cdots \cup M_m$ is still connected. Therefore, the set C_{1m} is a 1-connected m-dominating set. ☐

THEOREM 2. *The performance ratio of MDSA is $(5 + \frac{5}{m})$ for $m \leq 5$ and 7 for $m \geq 6$.*

PROOF. $|C_0| \leq |M_1| - 1$ which is proved in [14]. According to Lemma 1 and 2, the size of C_{1m} obtained from MDSA is bounded by $(5 + \frac{5}{m})|D_m^*| - 1$ for $k \leq 5$ and $7|D_m^*| - 1$ for $k \geq 6$. Since $|D_m^*| \leq |opt|$, where $|opt|$ is the size of the optimal (minimum) 1-connected m-dominating set, the performance ratio of MDSA is $(5 + \frac{5}{m})$ for $m \leq 5$ and 7 for $m \geq 6$. ☐

THEOREM 3. *The time complexity of MDSA is $O(m \cdot Diam)$ and the message complexity is $O(m(\Delta + 1)|V|)$, where Δ is the maximum node degree and $Diam$ is the diameter of the network.*

PROOF. The time and message complexities to build a CDS or MIS using the CDS-BD-D algorithm are $O(Diam)$ and $O((\Delta+1)|V|)$ respectively. Therefore, after adding the mth MIS M_m, the time and message complexities of MDSA become $O(m \cdot Diam)$ and $O(m(\Delta+1)|V|)$. □

5. DISTRIBUTED LOCAL DECISION ALGORITHM (LDA)

Although the DDA [13] algorithm is a distributed algorithm, the message complexity of DDA $O(\Delta^2|V|)$ is a little high especially for very dense networks. Considering collisions at MAC layer, distributed algorithms with low message complexities are preferred. Therefore, in this section we propose our distributed algorithm LDA whose message complexity is much smaller compared with DDA. In LDA, the colors of nodes are decided based on their local graph information. A set is a *common black node set* of v and u, denoted as $S_{cbn}(v,u)$, if $S_{cbn}(v,u)$ includes all the black nodes in $S_{cn}(v,u)$. Before we present our LDA algorithm, we firstly introduce an important Lemma 3.

LEMMA 3. *Given nodes u and v being neighbors, their closed neighbor sets are $N[u]$ and $N[v]$ respectively. For a node set P, $P \subseteq N[u]$, $v,u \in P$, $LVC(P)=k$ and a node set Q, $Q \subseteq N[v]$, $v,u \in Q$, $LVC(Q)=k$. If $|P \cap Q| \geq k$, then $LVC(P \cup Q)=k$. That means the graph superimposed by P and Q is k vertex-connected.*

PROOF. From Figure 2, if $|P \cap Q| \geq k$, there still exists at least one node in $P \cap Q$ after the removal of $k-1$ nodes from $P \cup Q$. And P and Q are still connected when those $k-1$ nodes are removed, since $LVC(P)=LVC(Q)=k$. Thus, the whole graph induced by $P \cup Q$ is still connected. Apparently, the graph superimposed by P and Q is k vertex-connected. □

Lemma 3 illumines us a distributed local decision way to make the whole graph k vertex-connected only using local graph information. Based on this Lemma, we propose our distributed algorithm LDA.

Our LDA has three phases:

1. Construct a 1-connected m-dominating set C_{1m} using MDSA.
2. Negotiate a common black node set: Every black node in C_{11} negotiates with its parents or siblings who are also in C_{11} about the S_{cbn} to make $|S_{cbn}| \geq k$. Since $|S_{cbn}| \geq 2$ and according to Lemma 3, this step is unnecessary if $k=2$.
3. Build a local k vertex-connected subgraph: Every black node in C_{11} builds a local k vertex-connected graph G_k which includes all the black neighbors in C_{1m} and S_{cbn}s, and marks all the nodes in G_k black.

Since we have already introduced MDSA in Section 4, we only focus on phase 2 and phase 3 here.

5.1 Phase 2: Negotiate a common black node set

In this phase, only the black nodes in C_{11} need to negotiate the common black node set with another neighbor which is also in C_{11}. A node is called an *attached node* if it is in C_{11} and finishes the negotiation procedure initialized by itself. Firstly, the root marks itself as an attached node and notifies its children. When a node v receives all the notifications from its parents and siblings with greater weights W, v starts this phase. If v can find a S_{cbn} whose size is no less than k and only contains the black nodes marked in C_{1m} between it and any attached node, then it is done. Otherwise, v should add some common white nodes into S_{cbn} to make the size of S_{cbn} no less than k with any attached node u. Then both of v and u mark all the nodes in S_{cbn} black. After finishing its negotiation procedure, v notifies its children and siblings. Repeatedly, all the nodes in C_{11} negotiate S_{cbn} with one neighbor in set C_{11}.

Algorithm 1 Build k vertex subgraph$(k, G(V,E))$

```
Sort nodes in non-increasing order in $G$ based on their $W(N_i, ID_i)$
Add all black nodes to $C$
while $C$ is not $k$-connected do
    Add a white node with the highest $W(N_i, ID_i)$ into $C$
end while
for every black node $u$ marked in phase 3 do ▷ Remove redundant nodes
    if $C$ is still $k$-connected after the removal of $u$ then
        Remove $u$ from $C$
    end if
end for
return $C$
```

5.2 Phase 3: Build a local k vertex-connected subgraph

After the negotiation phase, every node in C_{11} builds a local k vertex-connected subgraph G_k which includes all the neighbors in C_{1m} and the nodes in S_{cbn}s from its local graph using algorithm 1, and marks all the nodes in G_k black.

THEOREM 4. *The black nodes produced by LDA form a k-connected m-dominating set.*

PROOF. Select the first dominator node u which is the child of the root v and has a S_{cbn} with v, then add u and v to set H. All of u's marked neighboring nodes are added in set P and all of v's marked neighboring nodes are added in Q after u and v build their local k vertex-connected subgraph. According to Lemma 3 and $|S_{cbn}(v,u)| = |P \cap Q| \geq k$, $C = P \cup Q$ is k vertex-connected. Repeat this step, till all the marked nodes in C_{11} are in set H. The result C is at least k vertex connected. Furthermore, we build a 1-connected m-dominating set first. Then set C is a k-connected m-dominating set. □

THEOREM 5. *The performance ratio of LDA is $\max\{\frac{5}{m},1\}2\Delta$, where Δ is the maximum node degree.*

PROOF. After phase 1, we have $|C_{11}| \leq 2|M|-1$. The maximum number of nodes could be added to C for each black node v of C_{11} in phase 2 and phase 3 is $\Delta-1$, since there exists at least one black neighbor in $GL(v)$. After phase 3, the total number of black nodes in C is $|C| \leq 2\Delta|M|-\Delta$. According to Lemma 1, we have $|C| \leq \max\{\frac{5}{m},1\}2\Delta|opt|-\Delta$. Therefore, The performance ratio of LDA is $\max\{\frac{5}{m},1\}2\Delta$. □

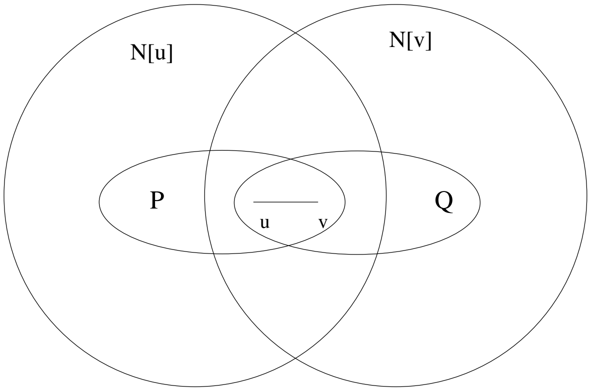


Figure 2: $LVC(P \cup Q) = k$ if $\mid P \cap Q \mid \geq k$ and $LVC(P) = LVC(Q) = k$.

THEOREM 6. *The message complexity of LDA is $O(\Delta|V|)$ and time complexity is $O((m+\Delta) \cdot Diam)$, where Δ is the maximum node degree and Diam is the diameter of the network.*

PROOF. In the negotiation phase, the message complexity can be bounded by $O(\Delta|V|)$. Therefore, the total message complexity is $O(\Delta|V|)$. The time complexity can be bounded by $O(Diam \cdot \Delta)$ in the negotiation phase. The total time complexity is thus $O((m+\Delta) \cdot Diam)$. ☐

6. CENTRALIZED ALGORITHM (ICGA)

If there is a base station or we know the topology of the whole network, we could use a centralized algorithm to obtain a smaller kmCDS. This is suitable for small-scale networks. Intuitively, the size of the obtained kmCDS is smaller than the one obtained from distributed algorithms. This conjecture could be verified by the simulation results in [13]. Almost all of the centralized algorithms mentioned in the literature try to find k-blocks or k-leaves for the purpose of showing the performance ratio. However, it costs much to find those k-blocks or k-leaves in a graph when $k \geq 3$ and this makes those centralized algorithms not easy to be implemented. The CGA algorithm proposed in [13] tries to deal with this from another aspect. In the augment step of CGA, the nodes with the maximum degree would be added to make the result set k-connected. Since there is no *matroid* [15] associated with this set, CGA cannot always guarantee obtaining a solution. Therefore, we propose a centralized algorithm ICGA to improve CGA. We also show the performance ratio of ICGA which CGA can not provide as well.

Before we introduce ICGA, we first illustrate two important lemmas.

LEMMA 4. *If G is a k-connected graph, and $G^{'}$ is obtained from G by adding a new node x with at least k neighbors in G, then $G^{'}$ is also a k-connected graph.*

This lemma has been proved in [16] by using separators.

LEMMA 5. *Given a k vertex-connected graph G and a connected set F which can k dominate G, the graph G' composed by $G \cup F$ is $k+1$ vertex-connected.*

PROOF. we can prove this Lemma by contradiction. Since G is k-connected, then after adding F, G' is still k-connected according to Lemma 4. Now we need to prove G' is $(k+1)$-connected. First, if G' is k-connected but not $(k+1)$-connected, then G' has a k-separator X. We examine two mutually exclusive cases from Figure 3.

- Case 1: $X \cap F = \phi$ which means $X \subset G$. Assume $T = \{T_1, T_2, \cdots, T_l\}$ are the connected components after X is removed from G. Since F dominates G, F must have a neighbor in each $T_1, T_2, \cdots, T_l$ and F is connected. Thus, T is connected by F, that means X is not a k-separator of $G' = \langle G \cup F \rangle$. From here, we can know that after removing any k nodes from G, G' is still connected.
- Case 2: $X \cap F \neq \phi$ which means X and F have some common nodes. However, F can k dominate G. After removing at most $k-1$ nodes from G, each node in F still has a neighbor in G and G is still connected since G is k-connected. Thus, G' is still connected and in this case X is not a k-separator of G'.

Therefore, X cannot be a k-separator and G' must be at least $(k+1)$ vertex-connected. ☐

Now, we introduce our ICGA algorithm (Algorithm 2) which consists of two steps:

1. Construct a 1-connected m-dominating set C_{1m}.
2. Augment set C_{1m} for k-connectivity sequentially according to Lemma 5.

Algorithm 2 also builds a 1-connected m-dominating set C_{1m} first. Here we use the centralized CDS construction algorithm CDS-BD-C2 [14] to construct a CDS C_{11}. Then C_{11} is augmented to become a 1-connected m-dominating set C_{1m}. Next, some nodes are added into C_{1m} to make C_{1m} k-connected sequentially. Here, a connected set F_i which can i dominate C_{im} is added for each loop according to Lemma 5. After C_{km} becomes k-connected, the redundant nodes are removed from this set to achieve a small size of C_{km} as [13] does.

THEOREM 7. *The set C_{km} derived from algorithm ICGA is a k-connected m-dominating set.*

PROOF. First, we build a 1-connected m-dominating set C_{1m}. In each *for* loop, we add a connected set F_i in which every node dominates at least i neighbors in C_{im}. According

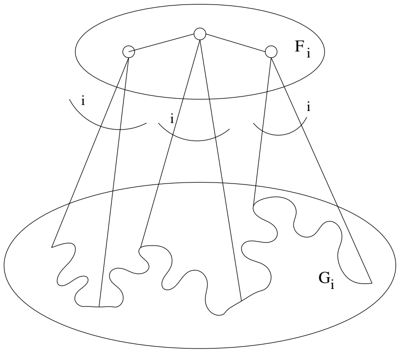


Figure 3: If G_i is i vertex-connected and F_i is a connected set and can i-dominate G_i, then $G_{i+1} = \langle G_i \cup F_i \rangle$ is $(i+1)$ vertex-connected.

Algorithm 2 ICGA($k, m, G(V,E)$)

```
1: Construct a CDS $C_{11}$ using CDS-BD-C2 [14]
2: Add all the nodes in $C_{11}$ to $C$
3: while There exits a white node whose number of dominator neighbors is less than $m$ do
4:     Put all the white nodes whose number of dominator neighbors is less than $m$ to $P$.
5:     Add a white node which dominates the largest number of white nodes in $P$ to $C$.
6:     $C_{1m} \leftarrow C$
7: end while
8: for $i = 1$ to $k-1$ do
9:     if $C_{im}$ is $(i+1)$-connected then
10:        $C_{(i+1)m} \leftarrow C_{im}$
11:    else                ▷ Augment $C_{im}$ to be $(i+1)$-connected
12:        Find a connected set $F_i$ with a small size which can $i$ dominate $C_{im}$
13:        $C_{(i+1)m} \leftarrow C_{im} + F_i$
14:    end if
15: end for
16: Remove redundant nodes from $C_{km}$
17: return $C_{km}$
```

to Lemma 5, after the ith loop, the graph induced by set $C_{(i+1)m}$ is $(i+1)$-connected. Finally, after $i = k-1$, set C_{km} is a k-connected m-dominating set. ☐

Theorem 8. *The performance ratio of ICGA is f, where*

$$f = \begin{cases} \begin{cases} 5k + \frac{5}{m} + 5H_{k-1} & m \leq 5 \\ 7k & m \geq 6 \end{cases} & k \leq 6 \\ \\ \begin{cases} 7k - 7 & m \leq 5 \\ 7k & m \geq 6 \end{cases} & k \geq 7 \end{cases}$$

and H_{k-1} is the $(k-1)$th harmonic number.

Proof. In order to bound the size of C_{km} obtained from ICGA, we should bound the size of F_i first. Actually, one can observe that F_i is a 1-connected i-dominating set of $C_{im} + F_i$ in each *for* loop. Since $|C_{im} + F_i| \leq |V|$ and $|F_i| \leq |C_{1i}|$, $|C_{km}| = |C_{1m}| + \sum_{i=1}^{k-1} |F_i| \leq |C_{1m}| + \sum_{i=1}^{k-1} |C_{1i}|$. According to Theorem 2, the performance ratio of ICGA is f. ☐

Theorem 9. *The time complexity of ICGA is $O(|V|^{3.5}|E|)$.*

Proof. It is obvious that the time complexity is dominated by step 2. First, we need $O(|V|^{2.5}|E|)$ time to check whether C_{im} is $(i+1)$-connected by using network flow techniques. In order to find F_i with the smallest size, we should find all the nodes which have at least i neighbors in C_{im}. Then we should remove the redundant nodes from F_i. Thus, $O(|V|(|V|+|E|))$ time is required. We also need $O(|V|^{3.5}|E|)$ time [13] to remove some redundant nodes in C_{km}. Therefore, the total time complexity of ICGA is $O(|V|^{3.5}|E|)$. ☐

7. DISCUSSION

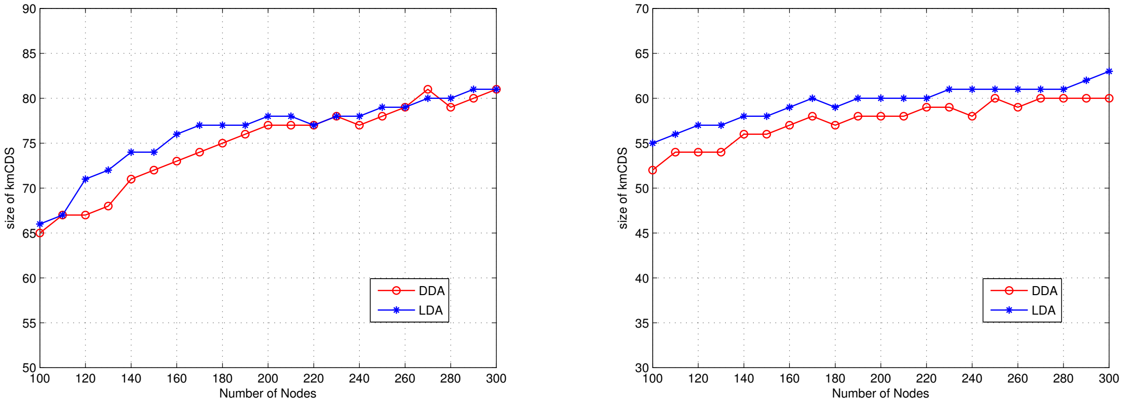
In this section, we discuss the performance ratio analysis of DDA which is given in [13] as $\frac{5}{m}(k^2+1)(m+42)$. We can have a tighter bound of the performance ratio of DDA if we use MDSA in phase 1 to construct a 1-connected m-dominating set.

Theorem 10. *The performance ratio of DDA is $(5 + \frac{5}{m})(k^2+1)$ for $m \leq 5$ and $7(k^2+1)$ for $m \geq 6$, if we use MDSA in phase 1 to construct a 1-connected m-dominating set C_{1m}.*

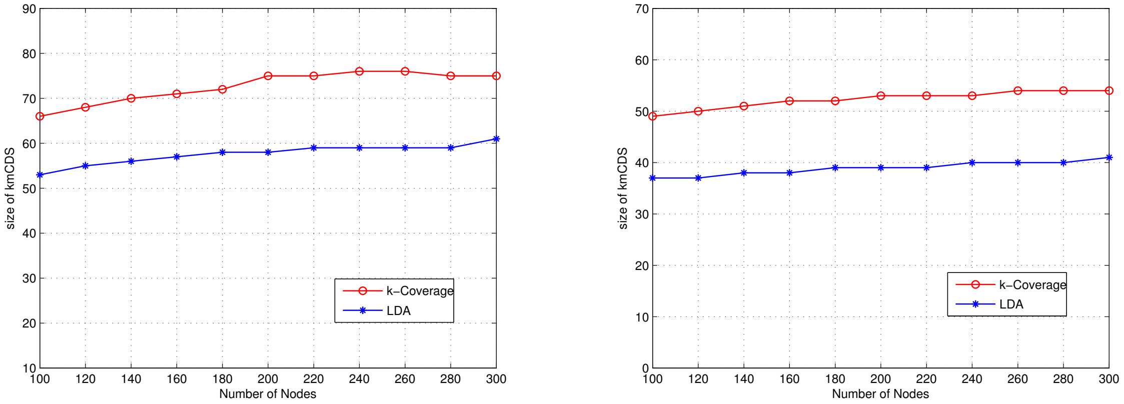
Proof. The number of connectors added to C is at most k^2 for each black node in C_{1m} in phase 2. After phase 2, the total number of black nodes in C is $|C| \leq (k^2+1)|C_{1m}| + (\Delta + 1)$, where $\Delta + 1$ is the size of the k vertex subgraph constructed by the root. According to Theorem 2, the performance ratio of DDA is $(5 + \frac{5}{m})(k^2+1)$ for $m \leq 5$ and $7(k^2+1)$ for $m \geq 6$. ☐

8. SIMULATIONS AND PERFORMANCE EVALUATION

We conducted simulations to evaluate our algorithms. In the simulations, nodes are randomly deployed in a $100m \times 100m$ square to generate various network topologies. In each of the simulations, all the nodes have the same transmission range which is $25m$. All of the data points were averaged over 100 simulation runs. We compared the proposed LDA algorithm with another two best distributed kmCDS construction algorithms from the aspects of the size of the obtained kmCDS and message complexity. Various simulation settings were tested in our simulations, and the results are very stable and similar for different settings. Due to space limitation, we only show the results for a particular setting as an example.

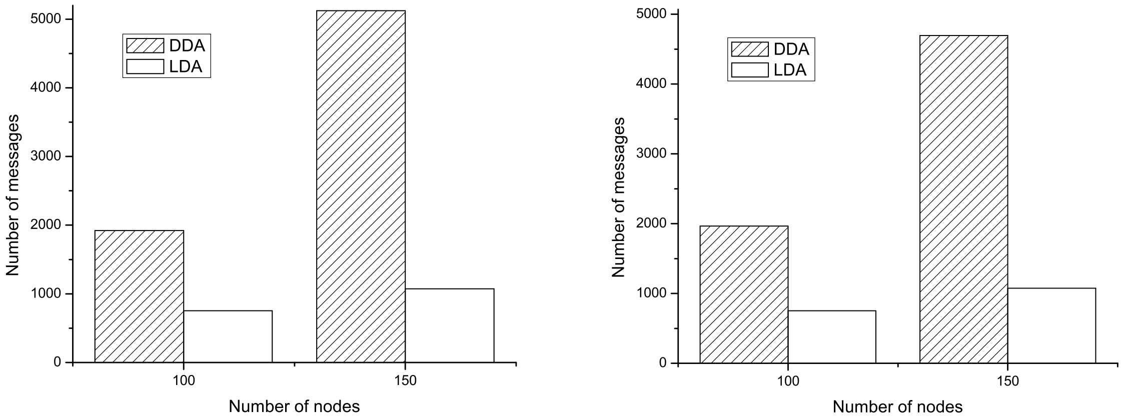


(a) $k = 4, m = 4$ (b) $k = 3, m = 4$



(c) $k = 3, m = 3$ (d) $k = 2, m = 2$

Figure 4: Comparison of the sizes of the obtained kmCDSs.



(a) $k = 4, m = 4$ (b) $k = 3, m = 4$

Figure 5: Comparison of the message complexities.

8.1 Size Comparison

In this group of simulations, we compared the sizes of the kmCDSs obtained by LDA, DDA and k-Coverage. Figure 4(a) and Figure 4(b) show that the size of the kmCDS obtained from LDA is a little bit larger than that of the kmCDS obtained from DDA. However, there is only a 5% difference between them. Considering the improvement on the message complexity by LDA over DDA, this compensation is acceptable, especially for large-scale networks.

We also compared LDA with k-Coverage [9] even though k-Coverage is only for the case where $k = m$. From Figure 4(c) and Figure 4(d), one can observe that LDA is better than k-Coverage. On average, LDA constructs a kmCDS whose size is 25.5% and 20.5% smaller for $k = m = 2$ and $k = m = 3$ respectively than k-Coverage does.

8.2 Message Complexity Comparision

We also compared the number of messages that are exchanged when using DDA and LDA. We only count the messages that are sent by the senders through all the phases, because the energy consumption of the senders is much more than that of the receivers. In order to compare these two algorithms fairly, we use MDSA to construct C_{1m} for both DDA and LDA. From Figure 5(a) and 5(b), it is obvious that LDA's message complexity is much lower than DDA's message complexity.

9. CONCLUSION

In this paper, we investigate the problem of constructing a k-connected m-dominating set in wireless sensor networks for general k and m. We propose one distributed algorithm LDA with low message complexity to construct a kmCDS whose size is guaranteed to be within a small constant factor of the minimum one when Δ is a constant. We also propose a centralized algorithm ICGA which has a constant performance ratio to improve the existing best centralized kmCDS construction algorithm CGA. Furthermore, we show the tight bound of the performance ratio of another existing distributed kmCDS construction algorithm DDA. There are some open problems left for our further study. The network model we used is static and symmetric. This problem will become much more complicated if we relax this network model to a mobile and asymmetric one. Therefore, it is of our interest to investigate the construction and dynamic maintenance of a kmCDS in a mobile environment.

10. ACKNOWLEDGMENTS

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